National Chen Kung University (Tainan, Taiwan) – ENSTA Bretagne (Brest, France)

Engineer assistant internship report

Optical temperature measurement: two-colour pyrometry

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Abstract

This document is a twelve-week long internship report in National Chen Kung University (Tainan, Taiwan) in the Department of Aeronautics and Astronautics for my teachers at my school ENSTA Bretagne. During this internship, I developed a two-colour pyrometry system (a device measuring temperature in an optical way) which is partially functional. After presenting the internship's location, this report will detail the underlying theory needed to understand how the two-colour pyrometry experiment works and will detail soot formation and a diffusion flame's structure. The calibration process and the verification on a propane partial premixed flame will be detailed. The best results will be presented, but unfortunately, they will not be convincing enough due to a lack of time to finish to set up the system; however ideas of improvements will be listed at the end of the report. The report will finish by an appendix including the MATLAB program used and specifications about the equipment used during this internship.

Keywords: two-colour pyrometry, optical, soot, temperature

Résumé

Ce document est un rapport d'un stage de 12 semaines à National Chen Kung University (Tainan, Taiwan) dans le département d'Aéronautique et d'Aérospatial et est adressé aux professeurs de l'ENSTA Bretagne. Durant ce stage, un système de pyrométrie bichromatique (technique optique de mesure de température) a été développé et est partiellement fonctionnel. Après une présentation du lieu du stage, ce rapport comportera une présentation de la théorie sous-jacente à la pyrométrie bichromatique ainsi qu'une description théorique de la formation de suie et de la structure des flammes de diffusion. Le processus d'étalonnage sera expliqué en détail, ainsi que l'expérience de vérification sur une flamme de diffusion. Les meilleurs résultats seront présentés mais ne seront malheureusement pas entièrement fiable dû à un manque de temps ; en revanche le rapport finira par présenter les nombreuses voies d'améliorations possibles pour rendre le système possiblement entièrement fonctionnel. Enfin, le lecteur pourra trouver en annexe de nombreuses informations complémentaires telles que les fiches techniques des équipements utilisés pour l'expérience ainsi que les programmes MATLAB développé durant le stage, ainsi qu'un dictionnaire Français-Anglais pour les termes scientifiques utilisés durant ce stage.

Mot-clés: pyrométrie, bichromatique, optique, température, suie

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Introduction

To end my second year in ENSTA Bretagne, a French post Graduate Engineering School specialized in Advanced Mechanics and Pyrotechnics delivering an engineer diploma after three years, I carried out my summer internship in the Department of Aeronautics and Astronautics (DAA) of the National Chen Kung University (NCKU) of Tainan in Taiwan between June 25th and September 17th, 2018. This twelve-week long internship aims to prepare students for their third and last year in ENSTA Bretagne and their final internship, leading to graduation. This internship in Taiwan was also my first experience of work in a foreign country, and I deliberately chose to do this internship in a research laboratory to discover the research world and the difference between the "classic" engineer world working in corporates. As the laboratory was specialized in many subjects including combustion, my goal was to develop during those three months a functional optical temperature measurement method. This report and the MATLAB programs would be useful for the DAA after I leave since no functional optical methods have been developed by students yet in this laboratory.

Temperature measurement of flames or in combustion chambers is essential for engineers to fully understand the combustion reaction and to meet the rising standards imposed on engines. One of the cheapest and simplest methods to measure temperature is by using thermocouples [9]; however, this method is not fully adapted to all the different situations an engineer will meet. Firstly, a thermocouple will not show the non-constant temperature in a flame or a combustion chamber as it only reads a local temperature at a given point. Secondly, turbulent flows and vortices will lead to highly unstable flames and these oscillations will totally false the measurement as it will give an average temperature at a spatial point and not a flame temperature. To some extent, the intrusive properties of a thermocouple measurement and soot particles emission will not only give incorrect results because of the soot deposition on the thermocouple probes (as it reduces heat transfer and therefore accurate temperature reading) but will also in some situations expose people to risks because of the highly toxic properties of soot particles and the elevated temperature and pressure. In those situations, optical techniques of temperature measurement must be considered as they are non-intrusive (or non-contacting) so the user does not have to be near the flame and will not be exposed to any risks mentioned earlier; it will also give a two-dimensional mapping of temperature, which is more appropriate for a flame or a combustion chamber, and the non-intrusive properties of optical methods also won't disturb the flame's aspect. Furthermore, optical methods may also estimate the soot concentration [1], which is highly important because of the carcinogenic properties of soot particles and their effect on global warming. [8][12][13]. However, this report will only focus on the temperature measurement part of the optical techniques.

Many methods or determining flame temperatures optically exists such as the Coherent anti-Stokes Raman scattering (CARS) [15], Laser Induced incandescence (LII) [16] or Tunable Diode Laser Absorption

Spectroscopy (TDLAS) [17], however two-colour pyrometry seems to be the simplest and more cost-effective method, while still being very accurate. Requiring only a CCD camera, a black-body radiation source and a computer, the two-colour pyrometry method seems to be the perfect optical method to develop during those twelve weeks of summer internship.

This report will present the conduct and the results of my twelve-week long internship. After a presentation of National Chen Kung University, the under-lying theory behind the two-colour pyrometry experiment will be explained, from soot formation to radiation theory equations needed to establish the MATLAB programs. Then, the experiment will be described, including the calibration process and the different results obtained with different programs and adjustments made. Results will then be discussed, and sources of improvement will be mentioned, as three months is a very short period to realize a fully functional pyrometry device. Finally, the report will finish with what I've learnt during this internship. If needed, the reader can have access to a French-English dictionary that can be found at the end of the report, translating all the scientific terms used during the internship.

I/ Description of the internship site

1) National Chen Kung University

National Cheng Kung University (NCKU) is a research-led and public university located in Tainan, Taiwan. It has more than 20,000 students and it is considered as one of the most prestigious universities in Taiwan and Asia (being considered as the 4th best university of the country). It was founded in 1931 under Japanese government as "Tainan Technical College" and became a national university in 1971. It is famous for having formed lots of notable Taiwanese personalities such as several ministers, highly-awarded scientists or architects and prestigious international university directors. Each year, new specializations are being created and new Departments are founded. The University also opened its formation worldwide and host several foreign students.

Figure 1: Tainan's localisation on a Taiwan map

NCKU is divided in 9 colleges: liberal arts, Social Science, Management, Sciences, Engineering, Medicine, Electricity & Computer Science, Planification & Design, Biology & Biotechnology. The Academics Program prepare students for Master Degree among its several departments. The Research Center has also a very strong place, with 15 different centers. Each of this colleges are separated in different departments for a total of 43 different departments [6].

- 1931 : Creation of the university as *Tainan Technical College*, located in Tainan
- 1942 : School renamed in Japanese during the occupation
- 1946 : School renamed *Taiwan Provincial College of Engineering* with 6 departments (Mechanical, Electrical, Chemical, Electro-Chemistry, Civil and Architectural Engineering)
- 1956 : School reformed as *Taiwan Provincial Cheng Kung University* with 4 different colleges gathering 10 departments
- 1962 : Master's Degree program in Chemical Engineering created
- Doctor's Degree program in Electrical Engineering created
- 1971 : School reformed as *National Cheng Kung University* ; Master's Degree program in Physics, Hydraulics and Ocean Engineering

Figure 2: National Chen Kung University

2) Economic analysis of the laboratory

ZAP Lab is a public research institute, mostly financed by the Ministry of Science and Technology (MOST). In 2018 the financial incomes provided by MOST were about 4,443,000NTD (about 125 100 euros), an amount increasing during the three last years (it was 2,614,000NTD in 2016 and 3,558,000NTD in 2017); this evolution is partly due to the growth of the university in the research field.

Today the laboratory has 20 employees including 16 masters, 2 PhDs, 1 professor and 1 assistant, spread in the different research projects. The laboratory also welcomes many graduated or non-graduated students achieving an internship or following their studies in the university.

The spending of the laboratory mostly concerns the acquisition of consumables, necessary to lead experiments (gas, materials), and of new equipment. The salary of employees is also considered and is distributed this way:

- Master: 6,000 NTD (about 169 euros)
- Assistant: 20,000 NTD (about 560 euros)
- PhD: 10,000 NTD (about 281 euros)
-

The laboratory must also provide the cost of foreign travels or project related expenses. To fulfil this budget, the laboratory receives funding from National Chung-Shan Institute of Science & Technology (NCSIST) and Metal Industries Research & Development Centre (MIRDC), other research units belonging to the government. Today, the policy of ZAP Lab is to develop its image worldwide, opening its door to foreign students or participating to international conferences and to give a better image of the research field to young engineers or students.

3) Conduct of the internship

Figure 3:inside the Department of Aeronautics and Astronautics (photo taken by Clémence Royer)

I was assigned in the "ZAP Lab" (Zic and Partners Laboratory) of the Department of Aeronautics and Astronautics under the supervision of Professor Yueh-Heng Li ("Zic") from June 25th to September 18th, 2018.

Created in 2014 by Professor Yueh-Heng Li, the ZAP Lab leads research subjects in micro-combustion systems, electric propulsion, thermophotovoltaic power systems, clean coal combustion technology and biomass energy. There were about 50 students in this laboratory. Most of them were master degree students, but there were also some PhD students and some interns from France, Thailand or India. The main studied subjects in the ZAP Lab were combustion experiments and electric engines for spacecrafts.

I worked from Monday to Friday from usually 9 AM to 5 or 6 PM, but since I had the keys to have access to the working room and the laboratory at any time of the day my working schedule was flexible, and I could also work on weekends. There was no cafeteria in the Department, so we had to eat outside; but many restaurants were surrounding the DAA, so eating did not lead to time loss. A room was available for the students and I worked there most of the time. I was also free to go to the combustion laboratory when I needed to do an experiment; some desks were available there, so I could work there a few days if I had to do experiments. However, the laboratory was sometimes crowded, and it frequently happened that I had to wait a day or two before doing an experiment, freezing the progress of my internship subject, but I wrote my report during those periods to compensate the time loss. I used English most of the time (for my report, reading scientific papers and communicating with other students from the laboratory); this internship made me improve my English level, and was also a perfect opportunity for me to practice my beginner level of Mandarin speaker.

During my internship, I was sometimes helped by some Taiwanese students who were very happy to help me (but often very busy, as they were preparing their master degree or PhD) and sometimes by my supervisor (but rarely as he was extremely busy); but since I was almost one of the only students to work (or have worked) on the two-colour pyrometry experiment, I couldn't get helped much by other students despite all their efforts to help me and I spent lots of days finding and reading scientific papers.

Every Monday afternoon from 4:00 PM to around 8:00 or 9:00 PM (sometimes later) we had a lab meeting with every students of the laboratory and the professors where 5 to 10 students (including interns) had to either present their subject progress or do a scientific paper presentation. The presentations were in English for the interns and in Mandarin or English for the Taiwanese students. I had the occasion to present my progress report twice during those twelve weeks: those moments were very important for my internship subject progress, as it was an opportunity to have help from every students and teachers from the laboratory by having different perspectives of the issues I got confronted to during the internship. It was also a good moment to know more about the other students' subjects or about a scientific research subject.

II/ Two-colour pyrometry experiment: Theory

The two-colour pyrometry method utilizes the thermal radiation at two different wavelengths from incandescent soot particles and measures its temperature; it therefore gives the temperature of the soot particles. However, it has been proven that the difference between the flame and the soot particles are negligible (< 1K) [2], so we can consider that this experiment measures the flame temperature.

In this part we will explain how the two-colour pyrometry technique will give us a temperature map using the radiation of the soot particles; we will however start by giving a description of a diffusion flame structure as we will be using mostly the two-colour pyrometry device on diffusion flames. We will also give some details about soot particles and their formation, essential for our experiment as they are primary source of a diffusion flame's luminosity [14].

1) Physical description of a laminar diffusion flame

As we use our two-colour pyrometry device on laminar diffusion flames, it is important to understand the structure of it. This part will give a brief physical description of a laminar diffusion flame. We will study here a laminar diffusion flame of a Bunsen burner in still air.

A diffusion flame contrasts a premixed flame (where oxidizer and fuel are mixed before flowing out of the Bunsen burner nozzle) as the oxidizer of a diffusion flame is on a regular basis the air surrounding the burner and only the fuel flows in the burner. The term "diffusion" comes from the fact that the fuel and the oxidizer will react together by diffusion: considering that the fuel flows along the flame axis, it will diffuse radially outward the central axis while the oxidizer (here the air) will diffuse radially inward. Fuel and oxidizer will react in the reaction zone (or the flame surface/flame sheet), where the temperature is higher than in the other parts of the flame. The reaction zone is thin (around 1 mm [14]) and does not "fill" the whole flame, therefore a diffusion flame can be considered as an annulus. It may also be important to notice on the figure below that on both side of the reaction zone, the mixture is either rich ($\phi>1$ inside the flame sheet) or lean ($\phi<1$ outside the flame sheet); therefore, a flame sheet can be defined as the locus of points where the equivalence ratio ϕ is equal to 1.

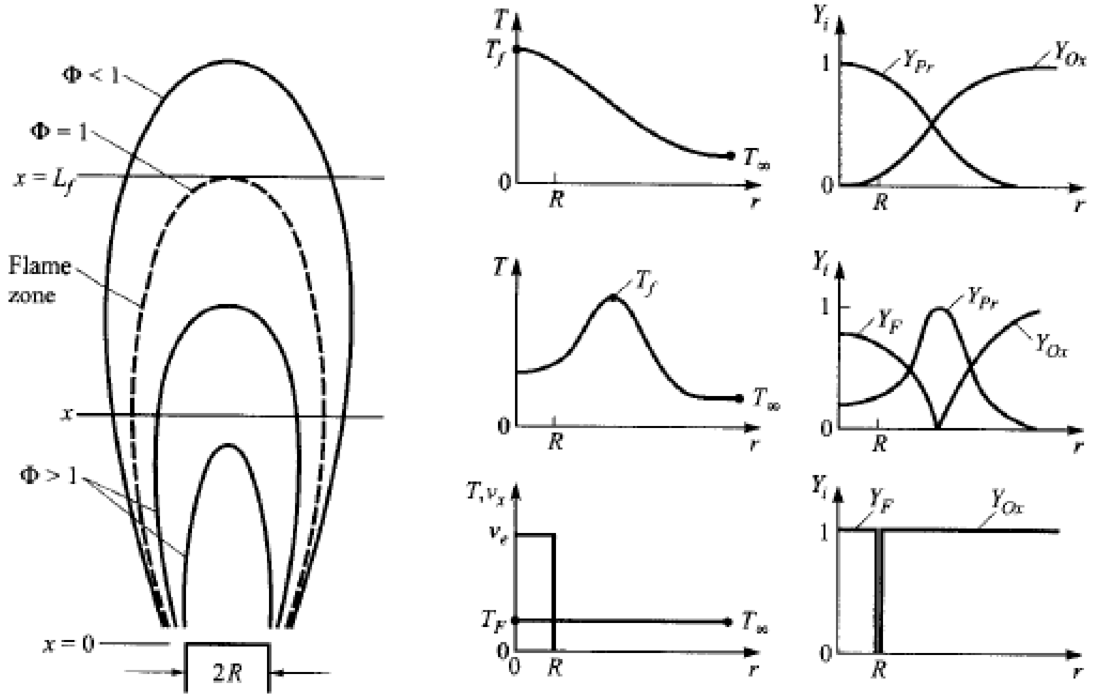


Figure 9.4 Laminar diffusion flame structure.

Figure 4:physical description of a laminar diffusion flame [14]

In the figure above (where T_F is the flame temperature, T_∞ is the ambient temperature, R the radius of the nozzle, v the velocity of the fluid and Y the mass fraction with the subscript corresponding to the fuel, the oxidizer or the flame products such as soot), we can observe radial profiles of different properties at different heights of the flame. The proportions are not respected for the readability of the image.

At the base of the flame (close to the nozzle) can be observed the potential core of the flame, a zone where the effects of viscous shear or diffusion can be neglected: in this zone the fluid properties (such as the velocity and the mass fraction) are uniform and are similar to the properties of the fluid inside the Bunsen burner ; however this zone is very small compared to the rest of the flame. In this zone, because of the lack of diffusion effects, oxidizer and fuel does not mix as seen on a graph, making a discontinuity in fuel and oxidizer mass fraction at the edge of the nozzle, and a constant temperature equal to the ambient temperature.

At the middle of the flame, the radial profile defines the reaction zone: the temperature gradient is very high around the flame surface, reaching its peak point (the flame temperature) at this zone. The reaction zone is where most of the flame products are made (some are created in the pre-heat zone, not visible in the figure) and can be observed at the highest mass fraction.

At the top of the flame, the graphs define the flame length: called L_f, it can be described as the height such that $\phi(r = 0, x = L_f) = 1$. For $r = 0$ and $x = L_f$ (corresponding to the top of the reaction zone or the flame tip), the temperature is at the flame temperature and it slowly decreases to the ambient temperature as r increases.

Above the flame tip, the amount of hot gas is enough and buoyant forces become important (as the gas temperature is high, the volume will increase therefore the density of the hot gas will decrease), accelerating the flow of gas and therefore narrowing the streamlines because of mass conservation, giving the typical shape of a flame.

2) Soot formation and destruction

Soot can be defined as the thick and black particles generated by an incomplete combustion (when there is not enough oxidizer in a combustion process) of hydrocarbon fuels. In a diffusion flame, there is always soot production and they characterize the yellow-orange colour of an incomplete combustion flame, contrasting the blue colour of a premixed flame (the typical blue colour results from excited CH radicals in the high-temperature zone within an excess of air [14]).

Because of the difficulty to study all the reactions occurring in a diffusion flame (due to a lack of efficient measurement method of the physical and chemical characteristics of soot during its development and evolution, but also because of computational limitations in simulations) [13], details about their formation and their chemical kinetics is still not fully understood and is subject to active research subjects, but we know that their formation is in concurrence with many other reactions such as formation of other hydrocarbons, hydrogen or carbon monoxide; in some milliseconds carbon and hydrogen atoms will cluster and form soot particles composed of millions of atoms in the reaction zone.

As seen of figure 5, the fuel will decompose into small radicals such as OH, O, H, CH and CH_2, which will lead to reactions that will generate larger hydrocarbon radicals and polycyclic aromatic hydrocarbons (PAH). PAHs, which are known to be precursors of soot formation and are being actively studied in the world, will nucleate, react, combine or grow to form incipient soot particles. These new particles, of around 1 to 6 nm wide, are mainly composed of carbon but contains a bit a hydrogen and may sometimes contain oxygenated species; they will then merge into larger particles of about 10 to 50 nm wide and will start to solidify, giving the soot particles. [13].

Depending on the situation, soot particles can be useful. Soot particles increase the energy loss in radiation and is useful for a candle as a sooty flame will produce more light than a non-sooty premixed flame; for a furnace it will increase radiative heat transfer. However, soot particles appear in the smoke and leave soot deposition which is undesirable for industrial, environmental or health reasons: its formation must be controlled as it can reduce the efficiency of an engine and increase the heat loss, have substantial adverse effects on cardiovascular and pulmonary health, and contributes to global warming. [8] [12] [13]

Soot particles are formed in the preheat zone on the fuel side of the reaction zone and most of them are consumed while flowing through the reaction zone, which is a highly oxidizing zone. However, some of them may not be oxidized and break through the flame, making what is usually called smoke.

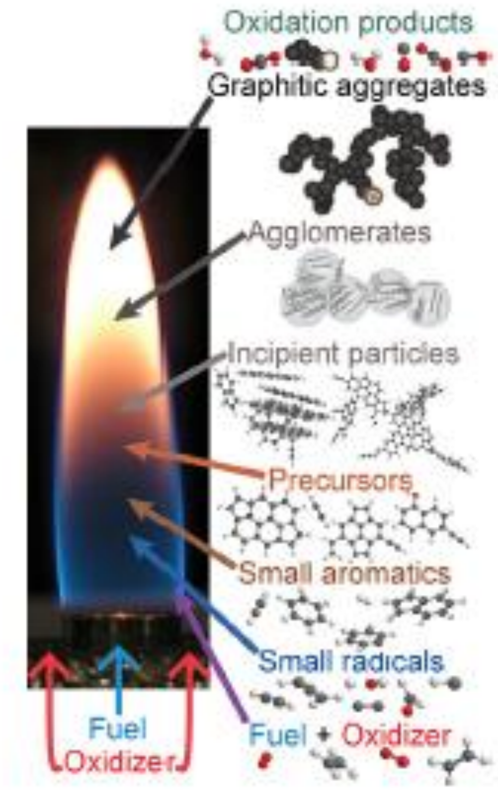


Figure 5: different steps of soot formation in a diffusion flame [13]

3) From emissive power to flame temperature

The theory of this method uses the thermal radiation theory. The first equation needed to understand the theory of the experiment is the Planck's equation, giving the monochromatic emissive power of a black body according to its temperature and wavelength:

$$I_{b,\lambda}(T) = \frac{C_1}{\lambda^5 [e^{\left(\frac{C_2}{\lambda T}\right)} - 1]} \quad (1)$$

Where:

- $I_{b,\lambda}$ is the spectral radiance of a black body at temperature T and at the wavelength λ ($W.m^{-3}$)
- λ is the wavelength (m)
- T is the temperature (K)
- $C_1 (= 2hc^2)$ is the first Planck's constant, with h being the Planck's constant and c the speed of light)

- $C_2 (= \frac{hc}{k_B})$ is the second Planck's constant ($= 1.4398 * 10^{-2}\ m.K$) , with k_B being the Boltzmann's constant)

However, soot particles cannot be considered as a black body. The monochromatic emissivity of a non-black body is defined as:

$$\epsilon_\lambda = \frac{I_\lambda(T)}{I_{b,\lambda}(T)} \qquad (2)$$

Where $I_\lambda(T)$ is the spectral radiance of a non-black body at the temperature T and at the wavelength λ.We can notice that ϵ_λ has no unit and is the fraction of the black body radiation emitted by a surface at wavelength λ.

For the two-colour pyrometry, we introduce a new quantity, which is the apparent temperature (or brightness temperature). Written T_a, it has no physical reality and is defined as the temperature of a black body that would emit the same radiation intensity as a non-black body at a given temperature T (or in other words, $I_{b,\lambda}(T_a) = I_\lambda(T)$). We can inject this new relation in (2), giving us a new equation:

$$\epsilon_\lambda = \frac{I_{b,\lambda}(T_a)}{I_{b,\lambda}(T)} \qquad (3)$$

We now have a ratio of two black body spectral radiances and can now combine (1) and (3): after simplification we obtain:

$$\epsilon_\lambda = \frac{e^{C_2/\lambda T} - 1}{e^{C_2/\lambda T_a} - 1} \qquad (4)$$

We can give another formula of the emissivity of soot particles by using the empirical correlation of Hottel and Broughton [3]:

$$\epsilon_\lambda = 1 - e^{(-\frac{KL}{\lambda^\alpha})} \qquad (5)$$

Where K is an absorption coefficient proportional to the number density of soot particles and L is the geometric thickness of the flame along the optical axis of the detection system; α is parameter that depends on the choice of several factors. The choice of the value of α will be described later.

If we combine (4) and (5), we obtain:

$$KL = -\lambda^{\alpha}\ln\left(1 - \frac{e^{\frac{C_2}{\lambda T}} - 1}{e^{\frac{C_2}{\lambda T_a}} - 1}\right)$$

K and L are difficult to find experimentally. The unknown product KL is needed to determine soot concentration [1]; however, since this report will only focus on the temperature measurement part of the two-colour pyrometry experiment, we can use two different wavelengths λ_1 and λ_2 to eliminate the product KL: by doing so, we obtain:

$$-{\lambda_1}^{\alpha_1}\ln\left(1 - \frac{e^{\frac{C_2}{\lambda_1 T}} - 1}{e^{\frac{C_2}{\lambda_1 T_{a1}}} - 1}\right) = -{\lambda_2}^{\alpha_2}ln\left(1 - \frac{e^{\frac{C_2}{\lambda_2 T}} - 1}{e^{\frac{C_2}{\lambda_2 T_{a2}}} - 1}\right)$$

Or formulated differently:

$$\left[1 - \frac{e^{\frac{C_2}{\lambda_1 T}} - 1}{e^{\frac{C_2}{\lambda_1 T_{a1}}} - 1}\right]^{{\lambda_1}^{\alpha_1}} = \left[1 - \frac{e^{\frac{C_2}{\lambda_2 T}} - 1}{e^{\frac{C_2}{\lambda_2 T_{a2}}} - 1}\right]^{{\lambda_2}^{\alpha_2}} \qquad (7)$$

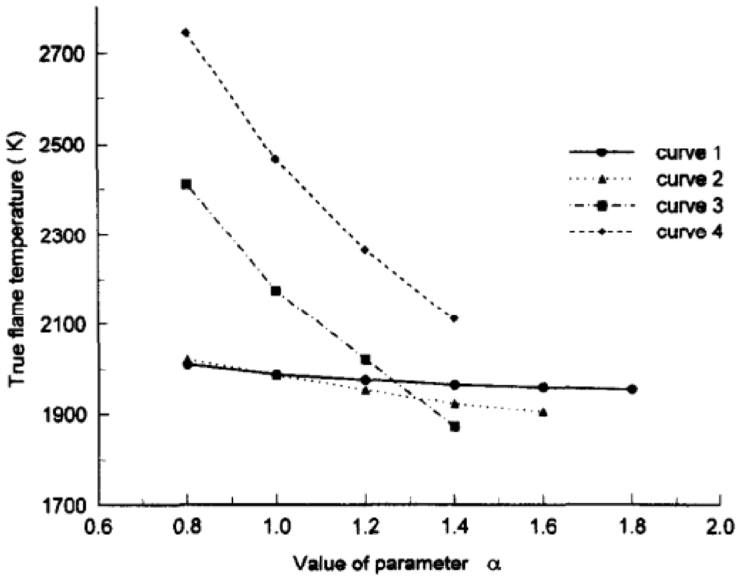
Since we can measure T_{a1} and T_{a2} experimentally, we can solve (7) and find T, which will be the flame temperature. It is important to notice that (7) has no literal solution therefore a numerical solver must be used to find the flame temperature. The value of α_1 and α_2 will be discussed under.

4) Selection of the two wavelengths and the value of α

As seen in the part above, the only missing data is the value of α_1 and α_2. H. Zhao and N. Ladommatos showed that these parameters depend on numerous factors, including the light wavelength, the soot particle size and the complex refractive index of the soot [1], which may be complex to estimate accurate values. However, they showed that the approximation α_1 =α_2 = α is accurate enough and can be chosen as a constant value under certain conditions.

As incandescent soot particles emit most of their radiations in the infra-red region [14], wavelengths in the infra-red regions are sometimes chosen for the two-colour pyrometry method.

In the graph below, we can observe the influence of the value of α on the estimated flame temperature for different chosen wavelengths: if the chosen wavelengths are in the visible domain (in this example, 550 and 750 nm), the estimated temperature is not highly sensitive to the choice of α (the estimated flame temperature will have variations of less than 50K for different values of alpha ranging from 0.8 to 1.8). However, if the two wavelengths are in the infra-red region, for the same range of α the measured temperature will vary from roughly 2800K to 2100K.



Wavelengths: curves1 and 2, 0.55 and 0.7 um (visible); curves 3 and 4: 2.5 and 4 um (infrared)

Apparent temperatures: curve 1, 1900 K and 1850 K; curve 2: 1700 K and 1640 K curve 3, 1400 K and 1050 K; curve 4, 1600 K and 1200 K

Figure 6: the effect of the value of alpha on the estimate flame temperature [1]

Therefore, the hypothesis of a constant value of α mentioned above is not valid if the two wavelengths are in the infra-red region: if wavelengths in the infra-red region are used, more information must be gathered to have more accurate values of α_1 and α_2. Moreover, the measuring system in the laboratory is much more sensitive to wavelengths of the visible region.

After all these considerations, we finally choose two different wavelengths: $\boldsymbol{\lambda_1 = 550\ nm}$ (green) and $\boldsymbol{\lambda_2 = 750\ nm}$ (red), both in the visible region of the light spectrum and **α=$\boldsymbol{\alpha_1}$ =$\boldsymbol{\alpha_2}$ =1.39**. The equation we will use in the programs to estimate the flame temperature becomes the following:

$$\left[1 - \frac{\frac{C_2}{e^{\lambda_1 T}} - 1}{\frac{C_2}{e^{\lambda_1 T a1}} - 1}\right]^{\lambda_1{}^{\alpha}} = \left[1 - \frac{\frac{C_2}{e^{\lambda_2 T}} - 1}{\frac{C_2}{e^{\lambda_2 T a2}} - 1}\right]^{\lambda_2{}^{\alpha}} \quad (8)$$

III/ Calibration of the optical device

1) Experiment description

The calibration part is necessary to have a relation between intensity and apparent temperature and therefore have an accurate flame temperature measurement. The experiment is described in the schema below.

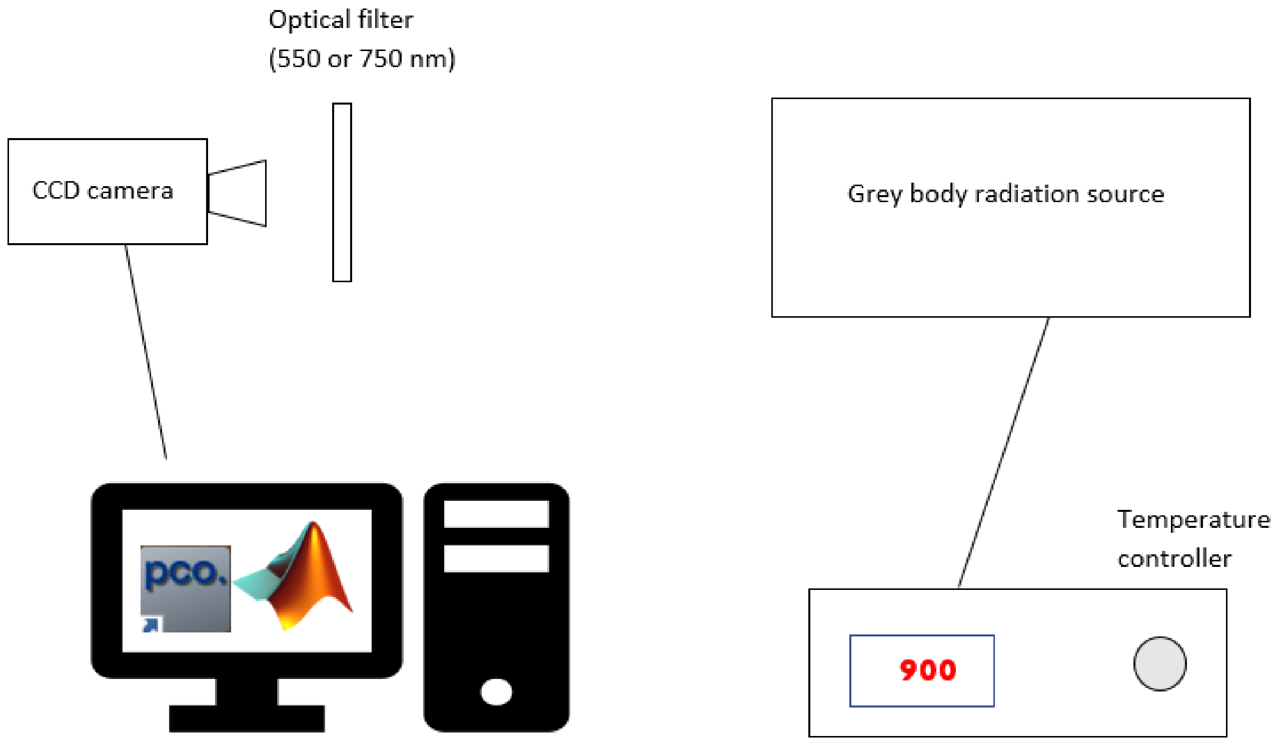


Figure 7: schema of the calibration process

To calibrate the optical device, we used a grey body radiation source, which is a special ceramic furnace[1], filmed in black and white by a CCD camera[2] linked to a computer. The temperature inside the furnace can be evaluated and controlled with a temperature controller linked to a thermocouple inside the furnace. We placed between the camera and the furnace a narrowband optical filter[3] relevant to the wavelength we chose for the experiment (550 et 750 nm). The CCD camera, aiming at the opening of the furnace, is linked to a software and allows us to take pictures of the opening of the furnace. The software used is Camware and will report the grey value of the different pixels of the picture in a .tif 8 bits file containing all the grey value of each pixel. It may be important to note that the camera was a 14 bits camera

[1] As the furnace was borrowed to a laboratory in Kaohsiung (south of Taiwan), the specifications about this furnace are unknown: however, it has a temperature range from ambient temperature to 1000°C with a resolution of 1°C.
[2] More information about the camera used can be found in the appendix B.
[3] More information about the optical filter used can be found in the appendix C.

(meaning that the grey value could go from 0 to 16 383). However, for the sake of the MATLAB programs we converted our .tif image into 8 bits image to have our grey value between 0 and 255.

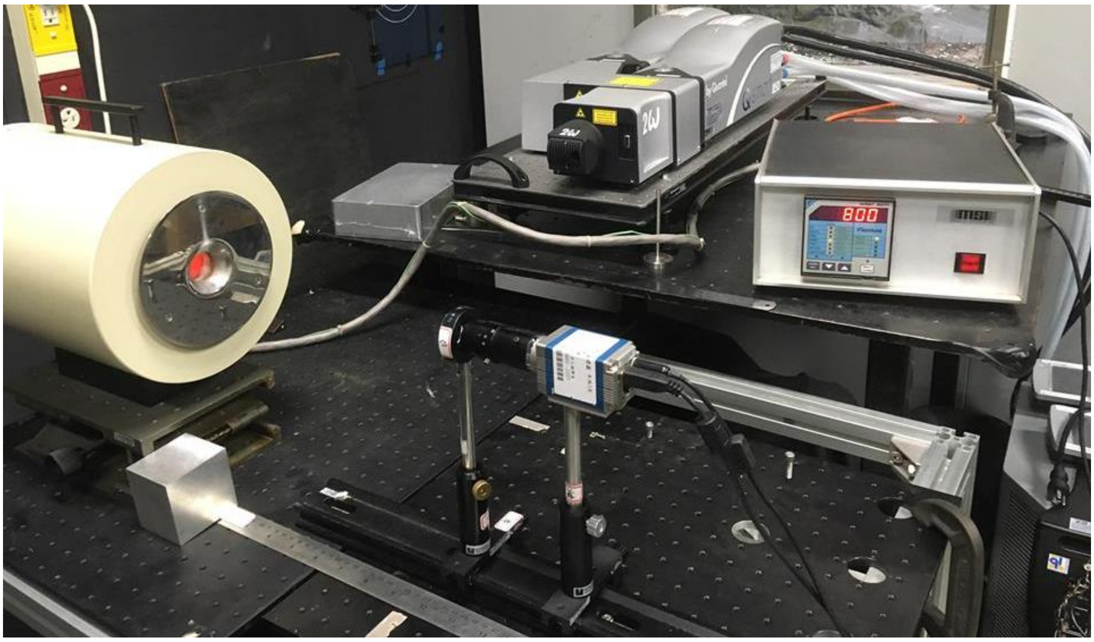


Figure 8: calibration of the optical device

The calibration process of the two-colour pyrometry system is divided in two parts: the first part is the experimental part where grey value information is gathered by taking pictures of the furnace, and the second is the elaboration of a mathematical relation between grey value and apparent temperature.

- The first part was made in the optical room of the laboratory and is the experiment described in the schema (figure 7). In a dark room, pictures of the inside of the furnace were taken at different temperatures (from 900 to 1000°C with one measure every 20°C, for a total of 6 measures) and with the two optical filters. With the software Camware, the pictures were taken and saved as 8 bits .tif files. 48 pictures were taken at each calibration process made during this internship: for each of the 6 different temperature steps, 4 different pictures of the furnace were taken and for both optical filters. This choice was made to make an average of the grey value and therefore have more accurate results. As the furnace had to be heated up from ambient temperature to 900°C then slowly to 1000°C, this part of the calibration process could easily take several hours.

- Once all the .tif files are saved, the second part of the calibration process was started using a computer. The .tif files were imported one by one on MATLAB as a 1392x1040 matrix (the dimension of the image in pixels) containing the grey value of each pixel of the picture. With the program grey_value_inside furnace.m (the program can be found in the appendix 5), an average grey value inside the furnace was reported on Excel (see figure below) and averaged with the mean grey value of the 3 other photos taken at the same temperature and with the same optical filter; this same process was repeated for each of the different .tif files.

	grey value :	
Temperature (°C)	λ=550 nm	λ=750 nm
900	4,7755	32,01015
920	5,18515	37,6399
940	5,715175	44,11195
960	6,6099	52,6619
980	7,699225	61,168175
1000	8,901625	69,53885

Figure 9: example of grey value data report

It has been shown [8] that a linear approximation can be done using this formula:

$$\ln(I) = b - a\frac{1000}{T},$$

where I is the intensity (or the grey value) of a pixel, T the temperature in Kelvin and a and b two positive constants. Knowing the temperature inside the furnace, we can draw a $\frac{1000}{T}$/ln(I) plot for the two wavelengths and have a sufficiently accurate formula between the temperature and the grey value for a black body: this linear interpolation gives us the value of T_{a1} and T_{a2} for each pixel. It will be recalled that unlike the grey body radiation source which can be considered as a black body, the soot particles cannot be considered as such: therefore, this formula cannot be used to estimate their temperature directly, but to find T_{a1} and T_{a2} which will help to solve the equation (8).

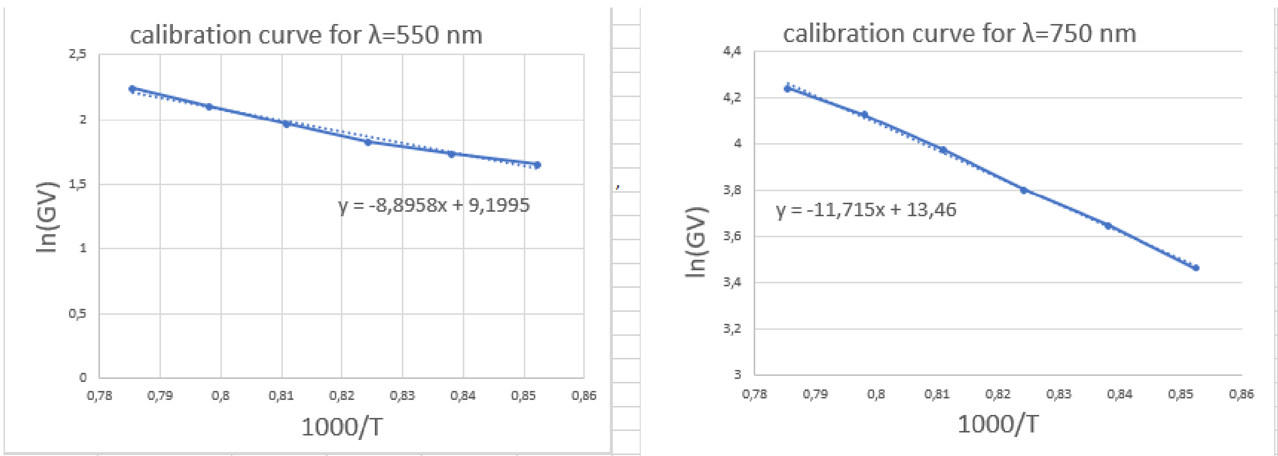


Figure 10: calibration curves for the two different wavelength and the equation of their linear approximation

The calibration of a such device can be tricky to do and took several tries during several weeks: after adjusting the height and distance of the different optical devices, other software parameters had to be fixed for the rest of the calibration part. After choosing a temperature range of 900 to 1000°C (with 6 measures taken, so one measure every 20°C), it was necessary to choose a good range of visible pixels (according to their grey value): if the range was not chosen carefully the picture would be too bright at the higher temperatures with the 750 nm optical filter (red filter), or the picture wouldn't be bright enough for the lower temperatures with the green optical filter. the flame brightness during the actual measurements (described in the part below) also needed to be taken in consideration: the flame was sometimes too bright (especially when using propane diffusion flame, which are very sooty therefore very bright), leading to an excess of white pixels and would skew the optical temperature measurement. The range chosen was finally between a grey value of 172 and 1950, leading to very dark pictures of the furnace, but the MATLAB programs were able to do the calibration with the pictures. Furthermore, we could also adjust the exposure time of the camera to change the amount of light received by the camera and therefore change the brightness of the pixel (the higher the exposure time, the brighter was the picture): an exposure time of 20 ms was chosen.

2) Sources of error

The calibration process was probably a source of lots errors in the estimations of flame temperatures. It requires to be extremely rigorous and careful, however despite all the efforts put in this part of the experiment, the conditions for the calibration weren't perfect: some dust could be seen on the optical filters, leading to dark spots on the pictures. As the optical filters are very difficult to clean, it probably skewed the formula between grey value and apparent temperature and therefore the flame temperature.

Furthermore, if the optical filter's normal vector was not totally lined up with the camera's line of sight (leading the a lightly tilted filter), a shade could be observed on the image, skewing the grey value of one side of the picture. After several tries, I couldn't remove the shade after testing different optical filter angles, but this issue was partially solved by averaging the grey value of the pixels in a zone of a picture not including the shade, but this lead to time loss. An exaggeratedly tilted optical filter (because the shade is often not easily visible) can be seen on figure below.

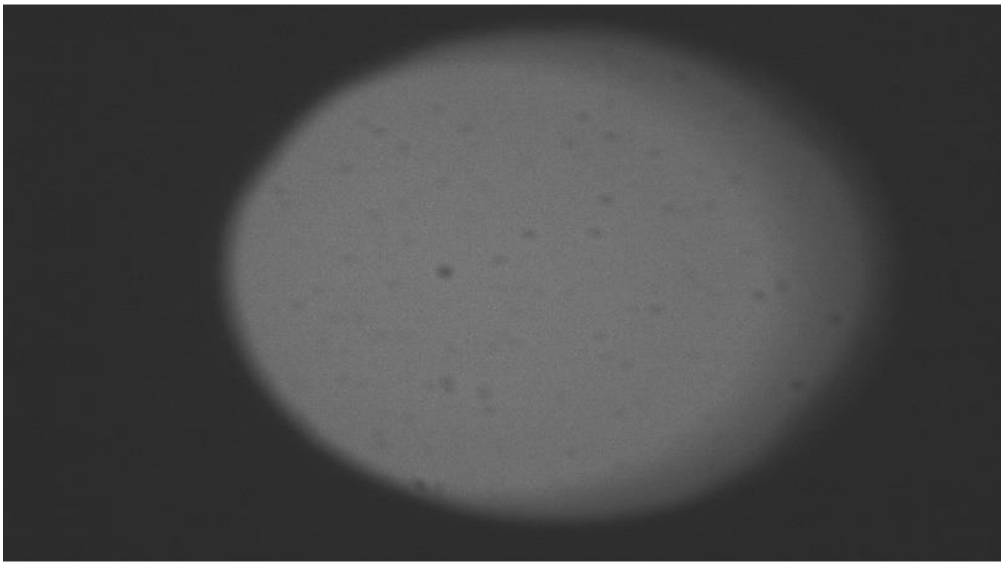


Figure 11: example of the shade (on the right side of the picture) and visible dust particles on the optical filter during the calibration process (750 nm optical filter; furnace temperature: 950 °C)

IV/ Trial run of the two-colour pyrometry system on a Bunsen Burner flame

1) Experiment description

After obtaining linear enough calibration curves, the optical system was tested on a flame in the optical room under the same condition as the calibration process. The goal of this experiment is to verify if the MATLAB programs were working or if the calibration was done correctly. To do so a MATLAB program (the programs are detailed in appendix F) would return a temperature field of the flame using the two-colour pyrometry method. The estimated temperature would then be compared to a temperature measured by a thermocouple placed in the flame at a known position. If there is a large discrepancy between the thermocouple-measured temperature and the optically estimated temperature is too high, the calibration process must be repeated, or the MATLAB programs must be readjusted.

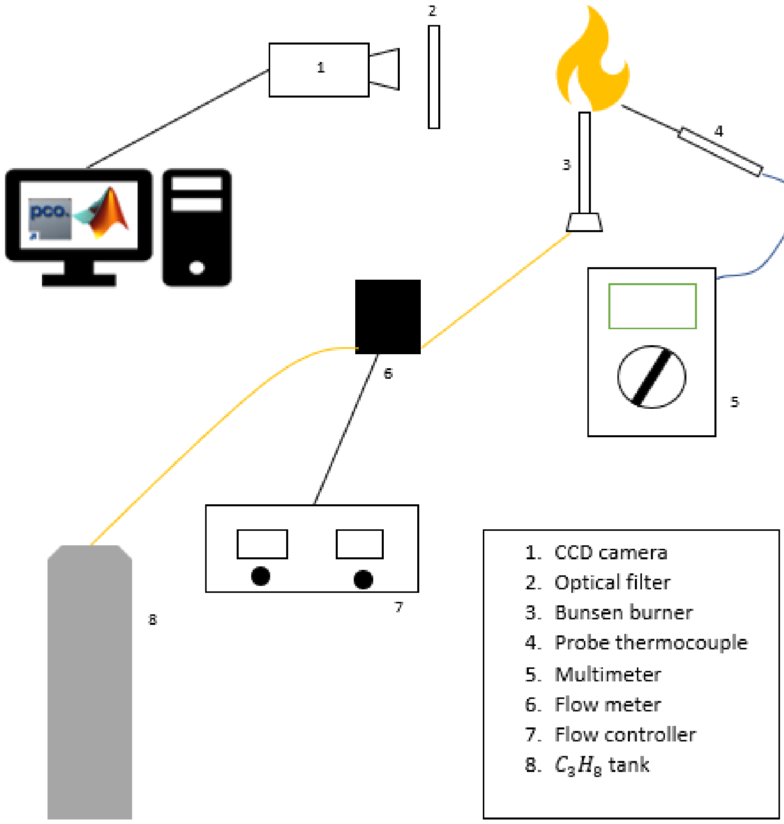


Figure 12: schema of the temperature measurement experiment

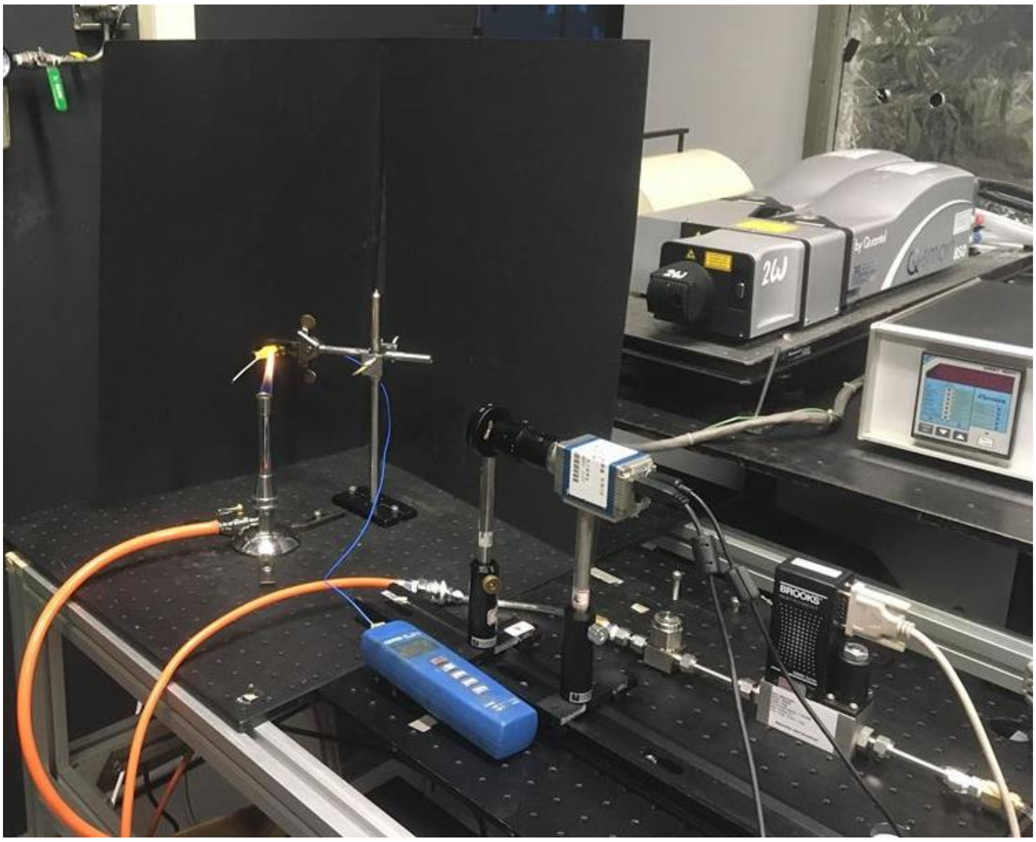


Figure 13: picture of the optical temperature measurement

The experiment is described in the schema seen above. A propane flame[4] from a 1.8 cm diameter Bunsen burner was made. The Bunsen burner was linked to a flow controller and flowmeter linked to a propane tank, so the flow could be controlled (an air tank was added later and is not represented in the schema above, but more details will be given under in paragraph IV-2-a). The two-colour pyrometry system (made of the CCD camera, the computer and the optical filter stand) was also installed on the table and in front of the flame to measure its temperature. Finally, a thermocouple was fixed on a stand near the flame. The thermocouple stand could be rotated to bring the thermocouple inside or outside the flame.

This experiment consists of several steps:

- Firstly, a first picture of the flame was taken with the thermocouple inside the flame. This picture was then saved and would give us the position in pixel of the position of the thermocouple's probe. The temperature of the point would then be measured; however, because of the oscillations of the flame the temperature measured by the thermocouple varied a lot therefore an average temperature over 30 seconds was calculated instead.
- The thermocouple was then removed from the flame for the optical temperature measurement: because of the elevated temperature, the thermocouple's probe would become hot and glow, resulting in a very bright light in the picture and skewing the temperature measurement.
- The lights were then turned off (to be in the same condition of the calibration process) and two pictures of the flame were taken: one with the 550 nm optical and another one with the 750 nm filter. The pictures (in .tif format) were taken carefully and nothing (except the optical filters) was changed or moved during the two pictures.
- Once the two pictures were taken, the two .tif files of the flame were imported on MATLAB and used by a program and would return a temperature field of the flame. The programs used will be described

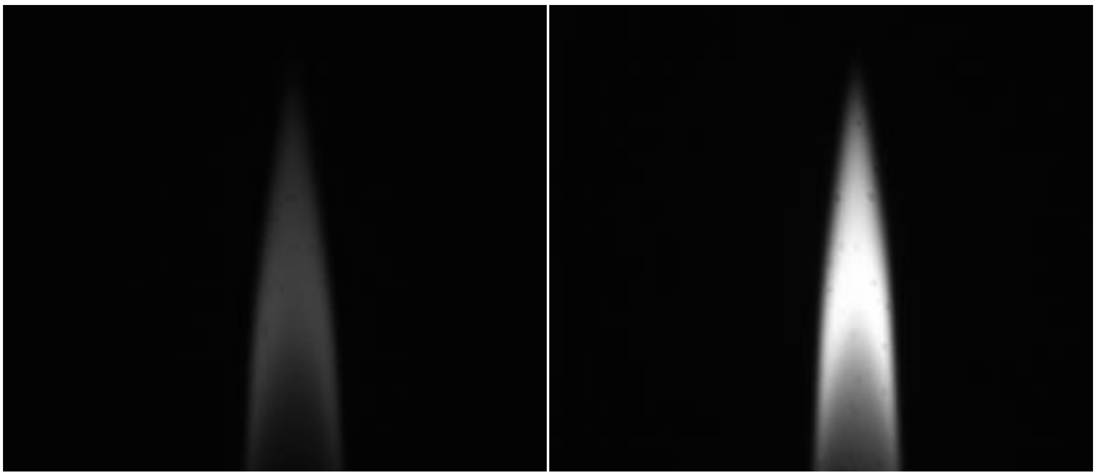


Figure 14: pictures of the propane flame (with the 550 nm optical filter on the left, 750 nm on the right). The picture represents about 1/3 to 1/4 of the whole flame

[4] The type of flame will be described in paragraph IV-2-a.

in detail in appendix F. The temperature in the thermocouple's probe area estimated optically was then compared with the temperature measured the thermocouple.

However, lots of adjustments were made to have the best results possible, but the principle of the experiment stayed unchanged. The adjustments are described in the part below.

2) Readjustments made

a. Choice of the flame: compromise between brightness and stability

The fuel and the type of flame was very important for the two-colour pyrometry temperature measurement experiment.

Firstly, the fuel used for the flame was a very important factor, as it played an important role in the brightness of the flame. A too bright or not bright enough flame would skew the measurement. For the first tries a methane (CH_4) flame was used to test the optical system. However, the pictures taken with the 550 nm optical filter were not bright enough to make an accurate enough temperature field. The fuel flow rate was increased to get a sootier flame (and therefore a brighter flame), making a bigger but much more unstable flame. As the stability of the flame is very important for this experiment (explained in the paragraph below), the idea of increasing the flow rate was forgotten. A more adapted change to do was to change the fuel and use propane (C_3H_8) instead of methane. As propane flames are sootier than methane flames, it was possible to have a small flame which are much more stable than higher flames and which would be bright enough for the two-colour pyrometry experiment.

Secondly, the stability of the flame was also a very important factor to consider. The two-colour pyrometry experiment uses two pictures of a hot source, taken with a different optical filter. The ideal set-up would need two cameras (one for each optical filter) taking the exact same picture at the same time. However, there was only one CCD camera in the laboratory, so the experiment had to be done with only one camera and the two pictures were taken in different moments. Therefore, it was important to have the most stable flame possible as the two pictures had to be identical pixel-perfect (in an ideal situation).

To do so, the first idea was to use a premixed flame to have the highest stability possible. This idea was abandoned rapidly because premixed flames are not sooty enough (therefore not bright enough) for the two-colour pyrometry experiment. A diffusion flame[5] was then made to have a much brighter flame (the difference in brightness can be easily observed in the figure below). However, the oscillations of the flame were still important even with a small flame. The final decision was to make a partial premixed flame with propane and air, with a very low flow rate of air: this was made by using a classic setup of a premixed flame (with one tank of propane and one tank of air, with the fuel and the air mixing before flowing through the Bunsen burner) but

[5] A physical description of a diffusion flame is given in appendix 7.

by adding another source of oxidizer with an opening on the Bunsen Burner. The low flow rate of oxidizer was not strong enough to turn the flame into the typical blue colour of a premixed flame but had an influence on the stability of the flame and would reduce the oscillations. The exact values of the flow rate could not be read on the flow controller since for this part of the experiment, the flowmeters used were not adapted for propane and air as they were already used by other students in another experiment or I couldn't find them. However, the exact flow rate is not important in this experiment as the goal was to control the flame to have a stable and bright one.

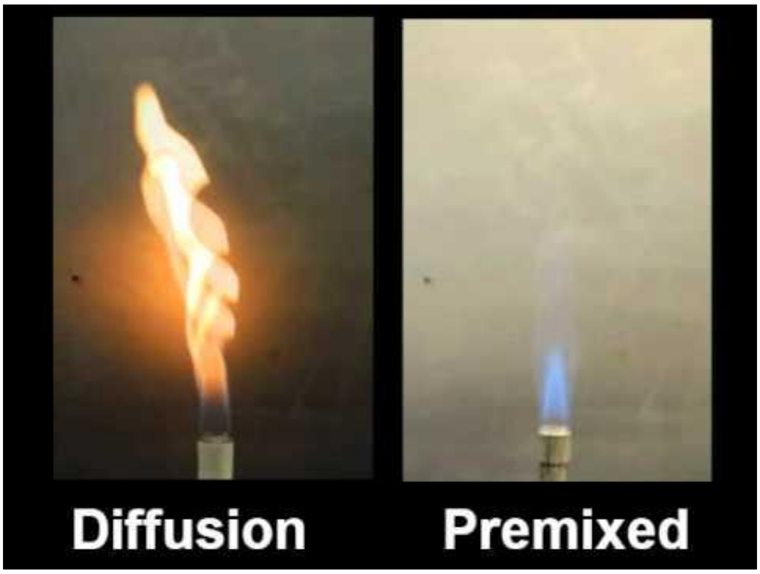


Figure 15: difference between diffusion and premixed flame (image from internet): the diffusion flame is brighter than the premixed one but much less stable

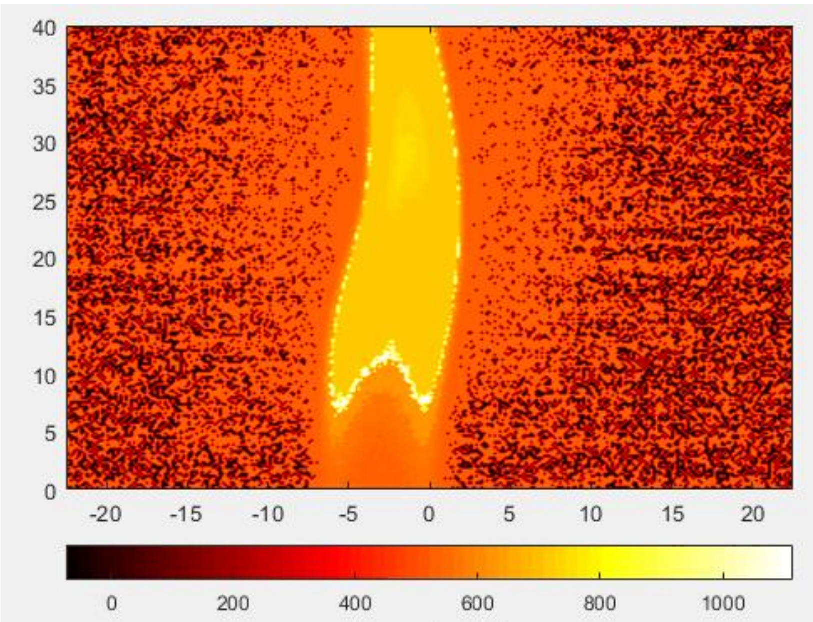


Figure 16: picture of one of the first tries: the non-stability of the flame and the too high brightness of the pictures skews the temperature field

Finally, other adjustments were made to improve the stability of the flame, as oscillations were still visible. Black cardboard panels (visible in figure 12) were installed near the flame to reduce the effect of airstreams, but also to reduce as much as possible a possible noise caused by background radiation . A quartz tube was initially placed above the flame to reduce the influence of the vortices making the flame flicker; however, the tube was broken, and the experiment had to be continued without it.

b. Choice of a correct thermocouple type

As the goal of this experiment is to verify if the two-colour pyrometry device is working, the choice of the thermocouple is important to have an accurate temperature measurement. More information about the principle of a thermocouple can be found in appendix D.

A K type thermocouple (Nickel-Chromium) was used for this experiment. It is one of the most used type of thermocouple because of its low price, its accuracy, its reliability and its wide temperature range (-270 to about 1372°C [7]). it was also chosen because the laboratory had all the raw materials to create my own thermocouple for the experiment.

Firstly, an industrial made thermocouple was used for the temperature measurement. The temperature recorded was very low (around 600°C) compared to the temperature estimated with the two-colour pyrometry experiment because the thermocouple had a protective sheath enclosing the thermocouple's probe. This cover is used to reduce the effects of corrosion on the thermocouple's probe and therefore increase its lifespan, but this sheath is not adapted to our experiment; it reduces heat transfer so is

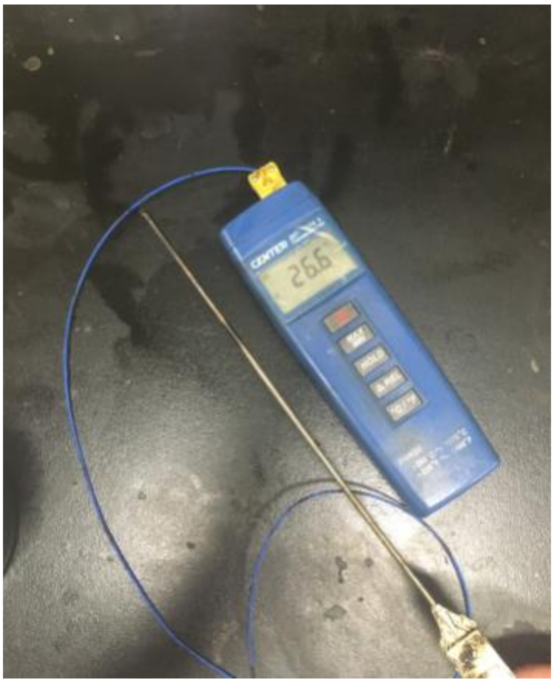


Figure 17: thermocouple with the protective sheath

Figure 18: raw home-made thermocouple (probe visible on the foreground)

much less sensible to temperature variations due to the oscillation of the flame, and its quite large size makes it not adapted to measure temperature at precise point, especially when we are near the reaction zone of the flame (which is about 1 mm wide [14]).A raw thermocouple was then created and used instead of the bought one: without the protective cover, the weld is directly in contact with the flame and its small size allows more accurate temperature measurement and was much more sensible to temperature changes. The average temperature of a point located on the edge of the flame was 971.6°C.

c. Changes in the MATLAB solver

Once the experiment has been optimized to have better results, the pictures can be taken and imported on MATLAB.

The first program used was temperature_map_vpasolve.m (the programs can be found in appendix F). this program will, from two pictures of the flame (in .tif format and taken with two different optical filters) use an implemented MATLAB solver to solve the equation (8) called *vpasolve,* which uses a Newton method to solve the equation, then will return a temperature field of the flame. As the solver was very slow, the pictures had to be reduced to 1/25 of their original size (as the program solves the equation (8) for each pixels of the picture, this reduced the solving time from 48 hours to a few minutes). The program returned the image below:

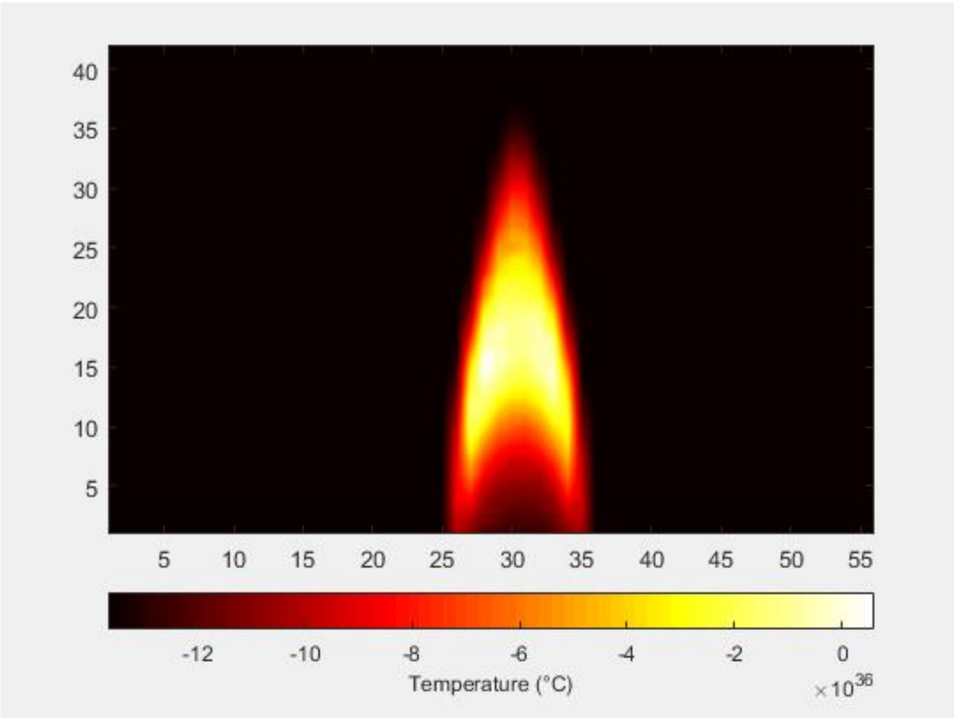


Figure 19: results from temperature_field_vpasolve.m

We can notice that the results are totally ridiculous, with negative temperature of -10^{36} K. After investigation, as the calibration pictures were taken carefully after several tries and the calculated apparent temperatures seemed to be correct, I find out that the problem came from the MATLAB solver *vpasolve* which is diverging. To deal with this issue, a MATLAB code called temperature_field_bisection.m using the bisection method was written to solve the following equation (which is just an adjustment of (8) to have a $\mathrm{f(x)} = 0$ problem):

$$\left[1 - \frac{\frac{C_2}{e^{\lambda_1 T} - 1}}{\frac{C_2}{e^{\lambda_1 T a1} - 1}}\right]^{{\lambda_1}^{\alpha}} - \left[1 - \frac{\frac{C_2}{e^{\lambda_2 T} - 1}}{\frac{C_2}{e^{\lambda_2 T a2} - 1}}\right]^{{\lambda_2}^{\alpha}} = 0 \qquad (9)$$

More details about the bisection method can be found in the appendix. As the program was much faster, the images were only reduced to 1/5 of their original size. After executing the program, we obtain this figure:

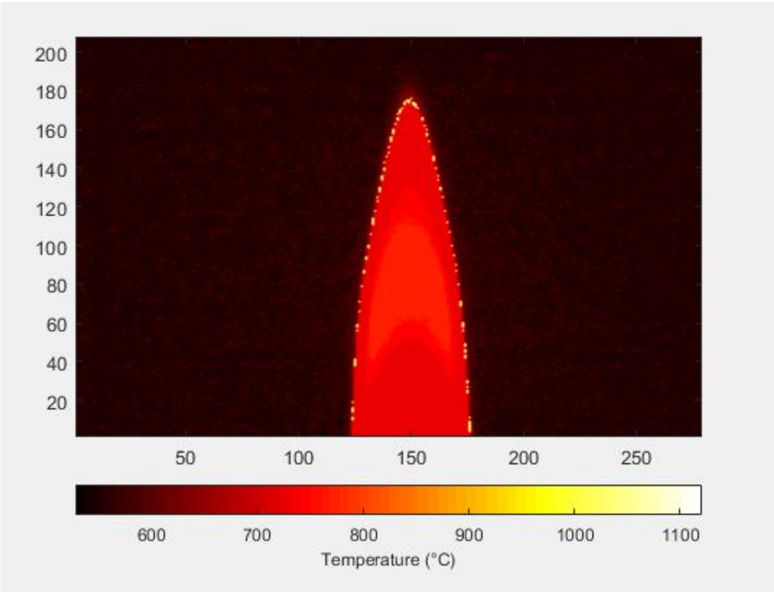


Figure 20: temperature field using the bisection method

3) Results analysis

The figure 20 seen above represents only the top part of the flame, which is about 1/3 or ¼ of the total flame's height.

The temperature field given by the bisection method program are better than with the *vpasolve* program, as the temperature range seems to be more realistic. However, the results aren't good enough: this program estimated a temperature of **726.9 °C** at the position where the thermocouple was and is far from the **971.6 °C** temperature measured by the thermocouple.

We can observe on the edge of the flame a dotted line with higher temperature than on the other parts of the flame (of around 1127 °C): in this hotter zone we can identify the reaction zone. However, the reaction zone should a continuous line enveloping the flame and not a dotted line like seen in the figure; furthermore, it’s temperature should be much higher than this (around 1600°C for a propane flame).

We can also notice that the ambient temperature is false: as there are no soot particles in the air and that this program estimates the temperature using soot radiation, it will give false results for non-sooty zones. Furthermore, the temperature inside the flame can also be discussed: as the picture is a line-of-sight picture of the flame and the flame reaction zone forms an annulus, a radial temperature profile of the flame might be more accurate: this will be discussed in paragraph V-3 concerning the Abel inversion method below.

Instead of questioning the validity of the temperature distribution given by the MATLAB programs, we can also question the accuracy of the temperature measurement. The temperature read by the thermocouple seems to be quite low for a propane flame: this may be due to the oscillations of the flame that are still present as it is a difficult issue to solve despite all the efforts made. To counter this issue, the two-colour pyrometry system was tested on a diesel spray combustion flame. The whole flame is much less stable, but the central axis of the flame is almost stationary. The temperature in this zone is known to stay constant over time and can be measured easily with a thermocouple. The idea was to try this system and only consider the temperature in the middle of the flame (as the temperature estimated in the outside zone would probably be false because of all the movements and the non-stationary physical properties).

Unfortunately, the diesel spray combustion flame was too bright for our two-colour pyrometry system: more details will be given in the paragraph V-1 below concerning improvements that can be made.

Figure 21: Test of the two-colour pyrometry device on a diesel spray combustion flame

V/ Source of improvement

As the results obtained by the two-colour pyrometry system created during those twelve weeks are not convincing enough, the system must still be improved; but twelve weeks is a short period of time to create a fully-working optical temperature measurement system. This part will mention all the possible ideas to improve the system and to make it fully functional and would be what I would try to do if I had more time.

1) Improvement in the used material

A major improvement that can be done to obtain better results would to have better or more equipment to realize the experiment.

Firstly, a better calibration source would be needed to realize the two-colour pyrometry experiment on brighter things than diffusion flames, such as on a combustion chamber or a spray combustion flame. The main problem of the laboratory's black body radiation source was its low maximum temperature of only 1000°C. as the calibration is done for a certain range of brightness for a fixed exposure time and visible grey value pixel range, if the brightness of the source is too high (for example with the diesel spray combustion flame), the calibration can not be done with the 550 nm optical filter (green) as the furnace is not bright enough. The furnace should go to at least 1500° or 2000 °C to maybe have enough brightness. Furthermore, a new black body radiation source would also have a specification manual that would contain important information for the students using this equipment. The calibration source can be a furnace or a tungsten lamp which is also adapted for the calibration of a two-colour pyrometry system.

A second CCD camera identical to the one used for the experiment would also be beneficial. As explained in the paragraph IV-2-a, a second camera would solve all the issues linked to the stability of the flame, which was in our case one of the most problematic and time-consuming problem. However, with two cameras the user would have had to write of MATLAB program to reconstruct the image as the pictures would not been identical.

On a less extent, the laboratory could also buy a spectrometer to obtain the spectrum of the flame and obtain more information about the intensity of each wavelengths. This idea is off-topic for this report as it is an expensive device (which deviates us of the main characteristic of the two-colour pyrometry method which is an inexpensive optical temperature reading method) and allows other optical methods such as the multi-colour pyrometry.

2) Changes in the post-experimental part

More time could've also been spent on the post-experimental part, meaning the computational part of the experiment.

Firstly, the MATLAB programs could have been improved. The program used a bisection method program to solve the equation (9), which is one of the most basic method of equation solving. Other methods such as the Lagrange method, or a home-made Newton method program could've been written to solve the equation and maybe obtain better results. Furthermore, the displayer of the temperature field could've been improved too: the program uses the *contourf* function of MATLAB to display the temperature field. This function creates a filled contour plot containing the isolines of the matrix *temp_field (*the matrix containing the temperature of each pixels of the picture*)* on the x-y plane (where the row and column indices of *temp_field* are the x and y coordinates in the plane, respectively). MATLAB automatically selects the contour lines to display, but we are free to choose the number of isolines to display (the isolines were then hidden for a more visible figure). Therefore, this program does not display the temperature of each pixel but only a fixed number of different temperature (which is equal to the number of isolines displays, which was chosen to 1000). The displayer could've been changed to display all the estimated temperatures.

Secondly, another way to verify the results of the two-colour pyrometry experiment would've been to realize a simulation of the experiment on STAR-CCM+. Propane and air flowmeters can be used to measure the flow rate of the gases and knowing the diameter of the Bunsen burner, the experiment can be simulated in the same condition as in the laboratory. This method is however (for STAR-CCM+ novice users) more time-consuming than verifying on a diesel spray combustion flame as the flowmeters also needs to be calibrated and the simulation done on computer.

3) Radial temperature distribution in a cross-section of the flame using Abel inversion

The two-colour pyrometry experiment set up during those twelve weeks allows us to obtain a line-of-sight temperature distribution. But it may also be interesting to obtain the radial distribution in a cross-section of the flame. As we consider the flame to be axisymmetric, a very effective method can be used to obtain the radial temperature distribution: the inverse Abel transform, or Abel inversion [18].

From a known line-of-sight integral projection data $p(y)$ (the line of sight temperature distribution), we can obtain the cross-sectional temperature field from the line-of-sight integrated image $f(r)$ using the inverse Abel transformation:

$$f(r) = -\frac{1}{\pi}\int_{y}^{R} \frac{dp(y)}{dy} \cdot \frac{dy}{\sqrt{y^2 - r^2}}$$

This one dimensional tomography method would be useful to determine the temperature inside the flame: according to the physical description of a diffusion flame given in paragraph II-1, a diffusion flame is an annulus (or can be considered as an empty shell), meaning that the soot particles in the middle of the flame may superpose and affect the global radiation in this zone, therefore maybe giving a false temperature inside. To obtain a cross-section temperature profile, an Abel inversion must be realized but due to the lack of time the program was not completed.

VI/ What I have learnt during this internship

This twelve-week long internship between my second year and my last year in ENSTA Bretagne made me discover for the first time a work experience in another country and an experience in a research laboratory. I learnt a lot during those three months.

Firstly, I learnt a lot of engineering theory during this internship. Even if most of the papers read were off-topic to what I will learn in my third year at ENSTA Bretagne, I still learnt more about combustion, flame structures, soot formation and radiation theory. Furthermore, I practiced my MATLAB coding and some combustion simulations on STAR-CCM+, even if no simulations were used for the report writing. On a less extent, I also improved my level in report writing and presentation (Microsoft Word and PowerPoint).

Secondly, this internship was highly beneficial for making my future work project clearer. I discovered what working in a research laboratory would look like, and this experience did not change my choice to work in a company as an engineer instead of in a research laboratory. But this internship made me change my vision of research laboratories: at first when I arrived here I was planning to do much more experiments and subjects than I did in reality. I did not expect the research laboratory life to be so slow: I was expecting to be guided, to find information quickly in research reports and would expect the experiments to go smoothly and have results quickly. But it was not the case; as I was working on a new subject in the ZAP Lab and because of the language barrier, it was difficult to be helped by other students and I was much more left alone than in other situation of project I have been in my life (for example school projects or my first-year internship in a company). I had to read a large amount of scientific report found in internet, but once again I was surprised as it would sometimes take me days of paper reading to find the answer to my questions, if I could find any answer online. And I was also surprised by how the experiment would go each time: it took sometimes days or even weeks before having a correct result because of all the key details and the rigour needed to conduct an experiment in the optical field. At the beginning of the internship I was sometimes getting frustrated and upset because I had the feeling that my subject wasn't moving forward for days; but when the day passed, I understood that everyone in the laboratory was in the same situation as me and my project progress went much better when I understood this, because I took more time to solve an issue and to think instead of putting myself too much pressure and trying to find a solution directly instead of taking time to understand fully the problem.

Finally, this internship made me change my work projects concerning the country where I would work. Before this internship, I was mostly considering working in France all my life. However, this internship being my first international work experience, I discovered how working abroad is interesting professionally and socially. I discovered the Taiwanese culture and way of life: they were the kindest, most respectful and friendliest people I have ever met. Working aboard gave me the occasion to discover a totally new culture and gave me a high interest in working abroad to discover new countries and meet new people.

Conclusion

As part of my second year in ENSTA Bretagne, I carried out a twelve-week long internship the Departments of Aeronautics and Astronautics of National Chen Kung University in Tainan, Taiwan under the supervision of Professor Yueh-Heng Li. During this internship, I was asked to develop from scratch a two-colour pyrometry device, a relatively cheap and viable temperature measurement device by an optical way. This device would be useful to the future students in the laboratory as no functional two-colour pyrometry was established yet here.

This complex experiment would reach theoretical knowledge in combustion, chemistry and radiation theory; it also required to code MATLAB programs and to realise some Star-CCM+ simulations for optimal results.

During this internship, the calibration was realised, and the device was tested on a partial pre-mixed propane flame. The results were however not convincing enough due to a lack of time and of optimal equipment: the experiment would require a better-working black body radiation source and possibly a second camera. More ideas of improvements are listed in the report to develop a fully functional two-colour pyrometry device.

This internship was highly beneficial for my future in a short term and also in a long term: firstly I learnt a lot about combustion theory and radiation theory, which would provide to be useful for my last year in ENSTA Bretagne and secondly it made me confident of my choice to work in an aerospace or aeronautic company instead of in a research laboratory when I graduate next year; however, it showed me how working abroad is interesting and fulfilling and made me change my initial plan of working only in France in the future.

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French-English Dictionnary

In this list can be found translations of English scientific words used during my internship. This mini-dictionary is also for the teacher of ENSTA Bretagne that will read my report.

Please note that some of these words are not in the report, but they were found in research papers used for this report and were useful to understand the underlying theory.

A
Annulus : couronne
Assumption : hypothèse

B
Bandwidth : bande passante
Biofuel : biocarburant
Bisection method : méthode de la dichotomie
Blend : mélange
Buoyancy : flottabilité (cf. poussée d'Archimède)

C
Calibration : étalonnage
Carcinogenic : cancérigène
CCD (Charge Coupled Device) camera : camera à transfert de charge
Chemical kinetics : cinétique chimique
Coarse : grossier (maillage)
Cross-section : plan de coupe d'un objet en 3D

D
Discrepancy : écart
Droplet : gouttelette

E
Emissivity : émissivité
Equivalence ratio : richesse de mélange (ϕ)

F
Furnace : four

G
Grey value : valeur de gris

H
Hydrocarbon : hydrocarbure

I
Incipient : naissant

J

K

L
Lean mixture : mélange pauvre
Line of sight : ligne de vue (d'une caméra)

M
Mean : moyenne (valeur)
Merge (to) : fusionner

N
Narrowband filter : filtre à bande étroite
Nascent : naissant
Noise : bruit (ex : d'un signal)
Nozzle : buse, embout

O

P
PhD (*Philosophiæ doctor*) : diplôme équivalent à un doctorat en France

Q

R
Refractive index : indice de réfraction

S
Software : logiciel
Soot : suie

Sootier : plus de suie (flamme)
Spectral radiance (or emissive power) : luminescence énergétique spectrale

T

Thermal flow meter : débitmètre thermique
Tilted : incliné, en biais
Trial run : période d'essai
Two-colour pyrometry : pyrométrie bichromatique

U

Uncertainty : incertitude

V

Viscous shear : contrainte de cisaillement (viscosité)
Vortices : tourbillons (pluriel de « vortex »)

W

Wavelength: longueur d'onde
Weld: soudure

X

Y

Z

List of figures

Appendix

Appendix A : internship supervisor final evaluation

ENSTA Bretagne

RAPPORT D'EVALUATION
ASSESSMENT REPORT

Merci de retourner ce rapport en fin du stage à :
Please return this report at the end of the internship to :

ENSTA Bretagne – *Bureau des stages - 2 rue François Verny - 29806 BREST cedex 9 – FRANCE*
☎ 00.33 (0) 2.98.34.87.70 - Fax 00.33 (0) 2.98.38.87.90 - stages@ensta-bretagne.fr

I - ORGANISME / *HOST ORGANISATION*

NOM / *Name* Department of Aeronautics and Astronautics, National Cheng Kung University

Adresse / *Address* No. 1, University Rd., Department of Aeronautics and Astronautics, National Cheng Kung University, Tainan, 70101, Taiwan

Tél / *Phone (including country and area code)* +886-6-2757575 ext63632

Fax / *Fax (including country and area code)* +886-6-2389940

Nom du superviseur / *Name of placement supervisor* Yueh-Heng Li

Fonction / *Function* Assistant Professor

Adresse e-mail / *E-mail address* yueheng@mail.ncku.edu.tw

Nom du stagiaire accueilli / *Name of trainee* Luc JOURDAIN

II - EVALUATION / *ASSESSMENT*

Veuillez attribuer une note, en encerclant la lettre appropriée, pour chacune des caractéristiques suivantes. Cette note devra se situer entre **A (très bien)** et **F (très faible)**
*Please attribute a mark from **A (very good)** to **F (very weak)**.*

MISSION / *TASK*

- La mission de départ a-t-elle été remplie ? A B C D E F
  Was the initial contract carried out to your satisfaction?

- Manquait-il au stagiaire des connaissances ? ☐ oui/*yes* ☑ non/*no*
  Was the trainee lacking skills?

  Si oui, lesquelles ? / *If so, which skills?*

ESPRIT D'EQUIPE / *TEAM SPIRIT*

- Le stagiaire s'est-il bien intégré dans l'organisme d'accueil (disponible, sérieux, s'est adapté au travail en groupe) / *Did the trainee easily integrate the host organisation? (flexible, conscientious, adapted to team work)*

  A B C D E F

Souhaitez-vous nous faire part d'observations ou suggestions ? / *If you wish to comment or make a suggestion, please do so here:* Luc has capability to self-learning and independence. He figured out complicate thermal radiation formula and developed the two color pyrometer measurement.

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Version du 03/05/2017

COMPORTEMENT AU TRAVAIL / *BEHAVIOUR TOWARDS WORK*

Le comportement du stagiaire était-il conforme à vos attentes (Ponctuel, ordonné, respectueux, soucieux de participer et d'acquérir de nouvelles connaissances) ?
Did the trainee live up to expectations? (Punctual, methodical, responsive to management instructions, attentive to quality, concerned with acquiring new skills)?

A [B] C D E F

Souhaitez-vous nous faire part d'observations ou suggestions ? / *If you wish to comment or make a suggestion, please do so here* ______

INITIATIVE – AUTONOMIE / *INITIATIVE – AUTONOMY*

Le stagiaire s'est –il rapidement adapté à de nouvelles situations ? A [B] C D E F
(Proposition de solutions aux problèmes rencontrés, autonomie dans le travail, etc.)

Did the trainee adapt well to new situations? A [B] C D E F
(eg. suggested solutions to problems encountered, demonstrated autonomy in his/her job, etc.)

Souhaitez-vous nous faire part d'observations ou suggestions ? / *If you wish to comment or make a suggestion, please do so here* ______

CULTUREL – COMMUNICATION / *CULTURAL – COMMUNICATION*

Le stagiaire était-il ouvert, d'une manière générale, à la communication ? [A] B C D E F
Was the trainee open to listening and expressing himself /herself?

Souhaitez-vous nous faire part d'observations ou suggestions ? / *If you wish to comment or make a suggestion, please do so here* Luc is open-minded to blend with Taiwanese student or foregin students here. They get well together.

OPINION GLOBALE / *OVERALL ASSESSMENT*

- La valeur technique du stagiaire était : A [B] C D E F
  Evaluate the technical skills of the trainee:

III - PARTENARIAT FUTUR / *FUTURE PARTNERSHIP*

- Etes-vous prêt à accueillir un autre stagiaire l'an prochain ?
  Would you be willing to host another trainee next year? ☑ oui/yes ☐ non/no

Fait à ______, le ______
In ______, *on* ______

Signature Entreprise ______ Signature stagiaire
Company stamp ______ *Trainee's signature*

國立成功大學
航空太空
工程學系
工學院

Merci pour votre coopération
We thank you very much for your cooperation

10

Version du 03/05/2017

Appendix B: CCD camera used for the two-colour pyrometry

pco.pixelfly usb I digital 14 bit CCD camera

technical data

image sensor

type of sensor	CCD
image sensor	ICX285AL
resolution (h x v)	1392 x 1040 pixel (normal) 800 x 600 pixel (center ROI)
pixel size (h x v)	6.45 µm x 6.45 µm
sensor format / diagonal	2/3" / 11.14 mm
shutter mode	global (snapshot)
MTF	77.5 lp/mm (theoretical)
fullwell capacity	16 000 e⁻ 24 000 e⁻ (binning)
readout noise	5 .. 7 e⁻ rms @ 12 MHz (typ.) 6 .. 8 e⁻ rms @ 24 MHz (typ.)
dynamic range	2 667 : 1 (68 dB) 4 000 : 1 (72 dB, binning)
quantum efficiency	62 % @ peak
spectral range	290 nm .. 1100 nm
dark current	1 e⁻/pixel/s @ 23 °C
DSNU[1]	2 e⁻ rms
PRNU[2]	< 1 %

general

power supply	9 .. 28 VDC (12 VDC typ.)
power consumption	< 4 W
weight	0.25 kg
operating temperature	+ 10 °C .. + 45 °C
operating humidity range	10 % .. 80 % (non-condensing)
storage temperature range	- 20 °C .. + 70 °C
optical interface	C-mount
CE certified	yes

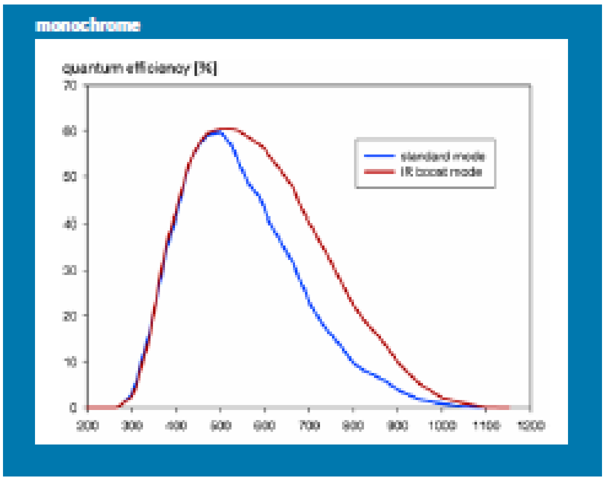
frame rate table

resolution pixelclock [MHz]	normal 12	24	center 12	24
1392 x 1040	7.3 fps	13.5 fps		
800 x 600			11.7 fps	21.6 fps
v2 binning	14.7 fps	27.0 fps	21.8 fps	40.4 fps
v4 binning	27.0 fps	47.0 fps	35.0 fps	62.0 fps

camera

max. frame rate	7.3 / 13.5 fps (12 / 24 MHz, normal) 11.7 / 21.6 fps (12 / 24 MHz, center)
exposure/shutter time	1 µs .. 60 s
dynamic range A/D	14 bit
A/D conversion factor	1.0 e⁻/count 1.5 e⁻/count
pixel scan rate	12 MHz / 24 MHz
pixel data rate	19.5 Mpixel/s
binning (hor x ver)	1 x 1 .. 4 x 4
non linearity	< 1 %
smear	< 0.002 %
anti-blooming factor	> 400 (standard 100 ms exposure) > 4 (NIR boost 100 ms exposure)
interframing time[3]	1 µs (optional)
trigger input signals	software / TTL level
trigger output signals	3.3 V LVTTL level
data interface	USB 2.0

quantum efficiency

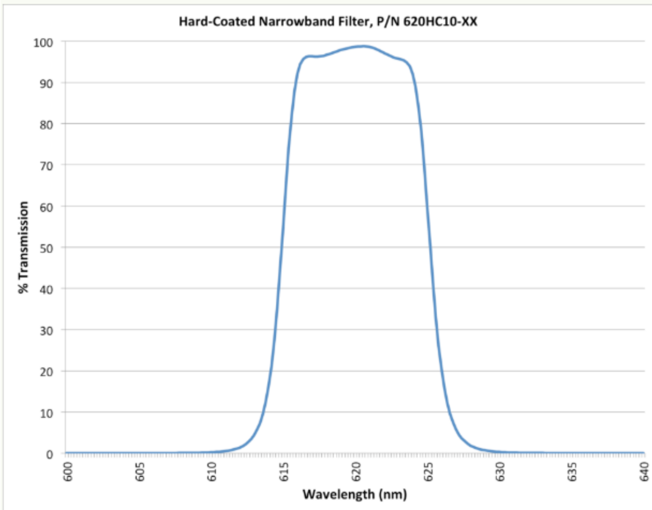


[1] dark signal non-uniformity measured in a 90% center zone of the image sensor
[2] photo response non-uniformity
[3] time between two consecutive images for particle image velocimetry (PIV) applications

Appendix C: Properties of the optical filter used for the two-colour pyrometry

General Specifications

Thickness:	5.0 ±0.1mm
Size Tolerance:	+0.0 / -0.1mm
Minimum Clear Aperture:	21mm dia.
Substrate Material:	Borosilicate Glass
Flatness:	3-5 waves
Surface Quality:	60-40 per MIL-C-48497A
Humidity and Abrasion:	Per MIL-C-675A
Durability:	Per MIL-C-48497A
Operating Temperature:	-50°C to +200°C
Mechanical:	Mounted in black anodized aluminum ring



Note: the graph shown above is not centred around the right wavelength. It is only an example of the general look of the transmission/wavelength curve of the optical filters we used.

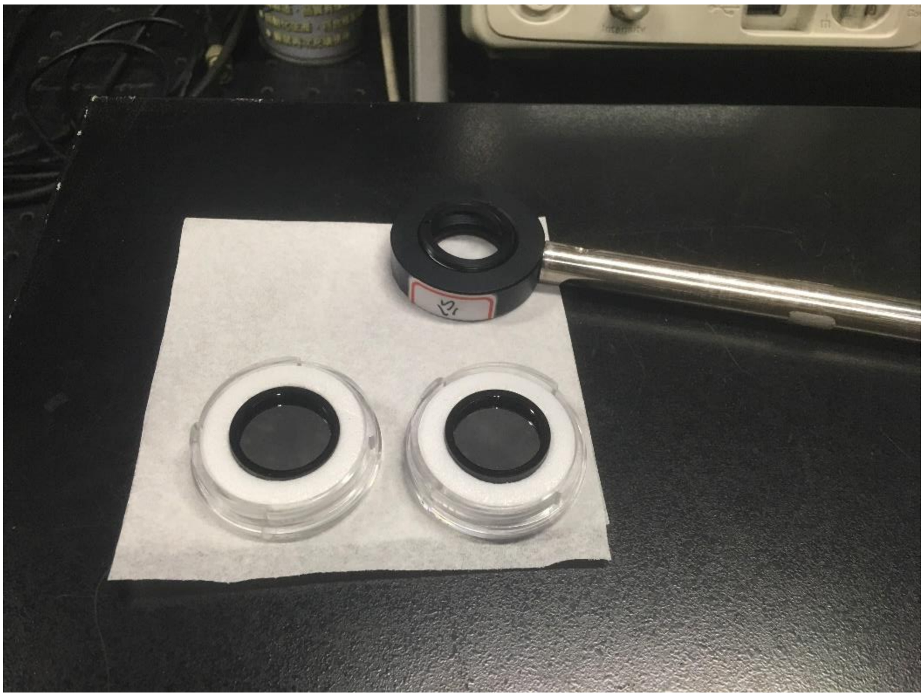


Figure 22: the two optical filters and the holder

Appendix D: How does a thermocouple works [7]

Thermocouples are sensors used to measure temperature. As they are extremely cheap, quite accurate and very versatile, they are commonly used in engineering applications or in research laboratories.

A thermocouple is composed of two wires made of different metals joined at both end by welds: one side is the measuring point (where the temperature will be measured) and the other is the reference point (where the temperature is known). By the Seebeck effect, when heating or cooling the measuring point a voltage is produced and can be directly correlated back to the temperature using this equation:

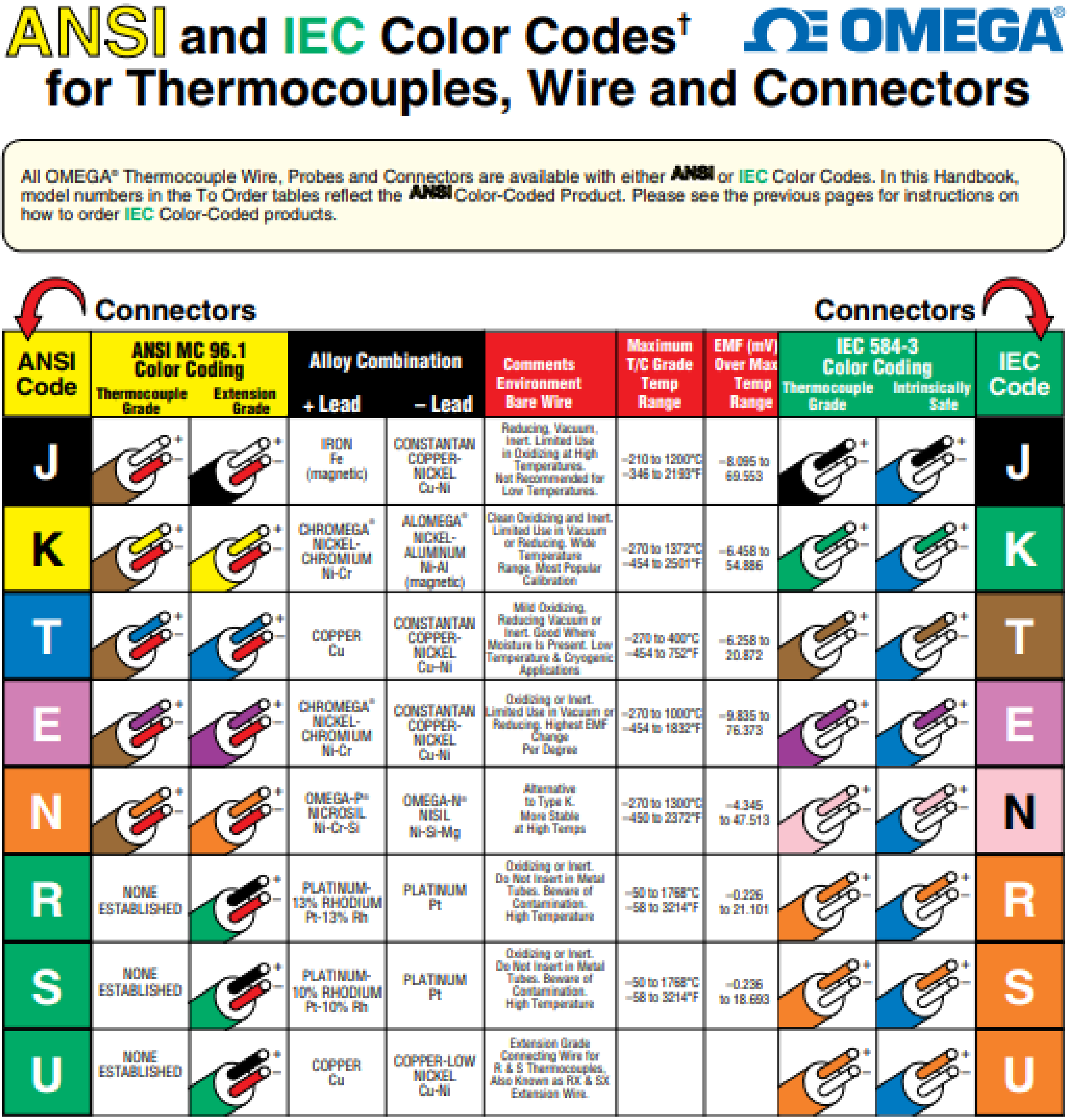
$$\overrightarrow{grad}(V) = -S(T)\overrightarrow{grad}(T)$$

Where $S(T)$ is the Seebeck coefficient and depends on the material property. If we integrate this equation, we obtain:

$$V = \int_{T_{ref}}^{T_{sense}} (S_+(T) - S_-(T))dT$$

Where $S_+(T)$ and $S_-(T)$ are the Seebeck coefficients of the conductors attached to the positive and negative terminals of the voltmeter respectively. With this formula, T_{sense} can be estimated.

Thermocouples are very versatile, but the right type of thermocouple should be chosen to have the best measurements possible. The efficiency of a certain type of thermocouple will depend on the temperature range and the chemical/abrasion/vibration resistance of the material or structure of the thermocouple. To find the right thermocouple, sheets are available on thermocouple selling websites [7].



ANSI and IEC Color Codes for Thermocouples, Wire and Connectors

OMEGA®

All OMEGA® Thermocouple Wire, Probes and Connectors are available with either ANSI or IEC Color Codes. In this Handbook, model numbers in the To Order tables reflect the ANSI Color-Coded Product. Please see the previous pages for instructions on how to order IEC Color-Coded products.

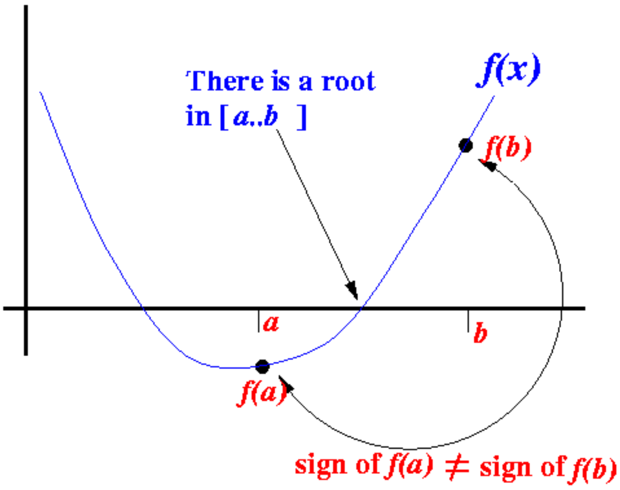
Connectors | Connectors

ANSI Code	ANSI MC 96.1 Color Coding: Thermocouple Grade	ANSI MC 96.1 Color Coding: Extension Grade	Alloy Combination: + Lead	Alloy Combination: – Lead	Comments Environment Bare Wire	Maximum T/C Grade Temp Range	EMF (mV) Over Max Temp Range	IEC 584-3 Color Coding: Thermocouple Grade	IEC 584-3 Color Coding: Intrinsically Safe	IEC Code
J			IRON Fe (magnetic)	CONSTANTAN COPPER-NICKEL Cu-Ni	Reducing, Vacuum, Inert. Limited Use in Oxidizing at High Temperatures. Not Recommended for Low Temperatures.	–210 to 1200°C –346 to 2193°F	–8.095 to 69.553			J
K			CHROMEGA® NICKEL-CHROMIUM Ni-Cr	ALOMEGA® NICKEL-ALUMINUM Ni-Al (magnetic)	Clean Oxidizing and Inert. Limited Use in Vacuum or Reducing. Wide Temperature Range, Most Popular Calibration	–270 to 1372°C –454 to 2501°F	–6.458 to 54.886			K
T			COPPER Cu	CONSTANTAN COPPER-NICKEL Cu-Ni	Mild Oxidizing, Reducing Vacuum or Inert. Good Where Moisture Is Present. Low Temperature & Cryogenic Applications	–270 to 400°C –454 to 752°F	–6.258 to 20.872			T
E			CHROMEGA® NICKEL-CHROMIUM Ni-Cr	CONSTANTAN COPPER-NICKEL Cu-Ni	Oxidizing or Inert. Limited Use in Vacuum or Reducing. Highest EMF Change Per Degree	–270 to 1000°C –454 to 1832°F	–9.835 to 76.373			E
N			OMEGA-P® NICROSIL Ni-Cr-Si	OMEGA-N® NISIL Ni-Si-Mg	Alternative to Type K. More Stable at High Temps	–270 to 1300°C –450 to 2372°F	–4.345 to 47.513			N
R	NONE ESTABLISHED		PLATINUM-13% RHODIUM Pt-13% Rh	PLATINUM Pt	Oxidizing or Inert. Do Not Insert in Metal Tubes. Beware of Contamination. High Temperature	–50 to 1768°C –58 to 3214°F	–0.226 to 21.101			R
S	NONE ESTABLISHED		PLATINUM-10% RHODIUM Pt-10% Rh	PLATINUM Pt	Oxidizing or Inert. Do Not Insert in Metal Tubes. Beware of Contamination. High Temperature	–50 to 1768°C –58 to 3214°F	–0.236 to 18.693			S
U	NONE ESTABLISHED		COPPER Cu	COPPER-LOW NICKEL Cu-Ni	Extension Grade Connecting Wire for R & S Thermocouples, Also Known as RX & SX Extension Wire.					U

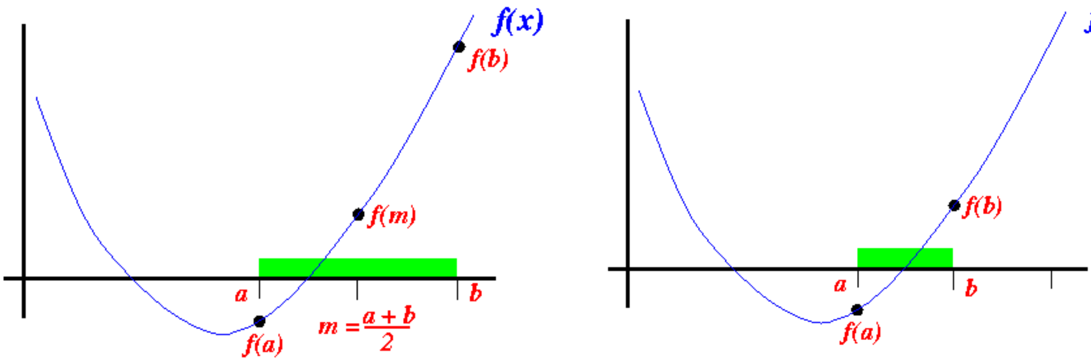
Figure 23:different type of thermocouple (OMEGA thermocouple)

Appendix E: bisection method

The bisection method is a numerical method to find a root of a given function f, which means x_0 such that $f(x_0) = 0$. This algorithm uses the intermediate value theorem, saying that if f is a continuous function on the interval [a;b] and f(a) and f(b) are of opposite signs, then there is a value x_0 ϵ [a;b] such that $f(x_0) = 0$. (the below images are from [10]).



The algorithm will check if the value of f at the extremity of the interval is equal to zero or not (or close to it). If a zero is not found there, the algorithm will split the interval in two segments [a;m] and [m;b] of equal length (with $m = \frac{a+b}{2}$), and will repeat the same steps on the segment where the values of f at the extremities are of opposite signs. The intervals will get smaller after each iteration and will converge to a root of the function. The user of the program can decide when the program will stop, depending on the minimal size of the interval or the proximity to a root.



This algorithm is one of the simplest to find a root of a function, however it has some downsides: the user must estimate a value of the root of the function. As it is to the user to decide in which interval the algorithm will start searching for the root, the initial interval [a;b] must contain the root or the program will not converge.

Furthermore, the user must be aware that if the initial interval contains several roots of f, the algorithm may not return the wanted root.

Appendix F: Matlab programs used in the two-colour pyrometry experiment

(1) Grey_value_inside_furnace.m

Editor - calibration_perso.m

calibration_perso.m

```
1 -   image=imread('E:\Luc two-colour pyrometry try 2\1000C\750_0004.tif');
2 -   [height width]=size(image);
3
4 -   inside_furnace=image(363:649,471:891);
5 -   mean(inside_furnace(:))
6
7
8
9
```

This program will monitor the grey value of each pixel and will show it on a 1392x1040 matrix called *image* (one grey value per pixel), where each grey value will be a number between 0 and 255. Then, the rest of the program will calculate a mean grey value of inside the furnace. Knowing the temperature inside the furnace and the optical filter used (here 1000°C with a 750nm wavelength optical filter), we can use this mean value to have a relation linking the grey value to the temperature of a black body.

It may be important to note that the line 4 of this program was modified later: we reduced each of our calibration picture to 1/5 of their original size (and therefore the position of the pixels inside the furnace was modified). As the programs to show the temperature map solved an equation per pixel, reducing the size of the images saved us a lot of time.

(2) Temperature_map_vpasolve.m

This program will plot the temperature map of a picture from two .tif files (one for each wavelength). After reversing the image (purely aesthetic part), it will for each pixel calculate the apparent temperatures T_{a1} and T_{a2} (in Kelvin) and solve the equation (8) to give the final temperature (in Celsius). Once the flame temperature of each pixel is estimated, it will plot it on a heat map.

```
image1=imread('D:\Stage 2A Taiwan\Photos CCD\verif 3\550_0001s.tif');       %image
taken with the 550 nm filter
image2=imread('D:\Stage 2A Taiwan\Photos CCD\verif 3\750_0001s.tif');       %image
taken with the 750 nm filter
image_550=image1(:,:,1);
```

```
image_750=image1(:,:,1);

[height width]=size(image_550);
reversed_image_550=zeros(size(image_550));      %optional, reverses the image to
get it in the right way
 for x=1:height
     for y=1:width
         reversed_image_550(x,y)=image_550(height-x+1,y);
     end
 end

 reversed_image_750=zeros(size(image_750));      %optional, reverses the image to
get it in the right way
 for x=1:height
     for y=1:width
         reversed_image_750(x,y)=image_550(height-x+1,y);
     end
 end

%assigning values to the different physical constants
C2=1.4388*10^(-2);
lambda1=550*10^-9;
lambda2=750*10^-9;
alpha=1.39;

temp_field=zeros(size(image_550));
k=height*width                                 %optional, indicates the number of
operations left to do in the loop below

 for x=1:height
     for y=1:width
         Ta1=(-9.4571*1000)/(log(reversed_image_550(x,y))-9.5825); %formula of
the Ta=f(GV) for 550 nm given by the calibration curves
         Ta2=(-11.77*1000)/(log(reversed_image_750(x,y))-13.497);  %formula of
the Ta=f(GV) for 750 nm given by the calibration curves

         e1= exp(C2/(lambda1*Ta1))-1;
         e2= exp(C2/(lambda2*Ta2))-1;
         y1=@(x) ((1-(exp(C2./(lambda1.*x))-1)./e1).^(lambda1.^alpha))-(1-
(exp(C2./(lambda2.*x))-1)/e2).^lambda2.^alpha;

         Tflame=bisection(y1,200,1800,1000,0.001,0.001);
         k=k-1
         temp_field(x,y)= Tflame-273.15;


     end
 end

 image_width=45;    %mm
 image_height=40;   %mm

[C,h] = contourf(linspace(-
image_width/2,image_width/2,width),linspace(0,image_height,height),temp_field,100
olormap(hot)
set(h,'LineColor','none')
c=colorbar('southoutside');
c.Label.String='Temperature (°C)'
```

(3) Temperature_field_bisection.m

This program is similar to the program above. The only difference is that instead of finding a numerical solution of (8) with *vpasolve*, it will solve the equation (9) by using a bisection method. The bisection method program is defined in another MATLAB file called bisection.m, visible below. Details about the bisection method can be found on appendix 4.

```
function [r] = bisection( f, a, b, N, eps_step, eps_abs )

    if ( f(a) == 0 )
    r = a;
    return;
    elseif ( f(b) == 0 )
    r = b;
    return;
    elseif ( f(a) * f(b) > 0 )
        error( 'f(a) and f(b) do not have opposite signs' );
    end

    for k = 1:N

        c = (a + b)/2;

        if ( f(c) == 0 )
            r = c;
            return;
        elseif ( f(c)*f(a) < 0 )
            b = c;
        else
            a = c;
        end

        if ( b - a < eps_step )
            if ( abs( f(a) ) < abs( f(b) ) && abs( f(a) ) < eps_abs )
                r = a;
                return;
            elseif ( abs( f(b) ) < eps_abs )
                r = b;
                return;
            end
        end
    end

    error( 'the method did not converge' );
end
```